Geometry, Architecture, Shapes and Patterns

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INTRODUCTION

Community Services is an alternative school in the Houston Independent School District. Students served by Community Services have physical, mental or emotional disabilities that require the provision of educational services in settings other than the normal school classroom. These educational options are available to qualifying students from early childhood through the age of twenty-one. Also, the students may come from the various schools in the Houston Independent School District (HISD), other school districts in the state of Texas, other states and foreign countries.

Community Services consists of three major components: agencies, hospitals and Homebound. First, agencies serve students who have severe physical, mental or emotional needs. Students suffering from child abuse or drug abuse would be examples of students requiring services outside the regular school setting. Second, hospitalized students as outlined in the “Community Services Handbook” whose medical program includes the provision of educational services are taught by qualified teachers in the hospital setting. Special consideration is given to the student’s physical, mental and emotional limitations. Third, Homebound instruction is available for regular and special education students who live within the boundaries of HISD and are expected to be out of school for four consecutive weeks or longer due to health reasons as documented by a physician. This continuation of studies ensures a smooth transition back to the “home” school when the student is ready to return.

I teach in the homebound component of Community Services. All of my students are ARDed into the homebound program. The Admission, Review, and Dismissal (ARD) committee is composed of a student's parent or parents and school personnel who are involved with the student. The ARD committee meets at least once a year to determine a student's eligibility to receive special educational services and to develop, review and revise the student's Individual Education Plan (IEP).

The IEP is a written plan designed for the individual student, which documents the services, the student needs, how they will be provided, and how progress will be measured. While not as detailed as a teacher's lesson plan, it must contain annual goals and the objectives needed to reach each goal. It short, it is an agreement between the school and parents on how the child will be educated. If not a Special Ed student, he or she is given a temporary Special Ed label until the end of services. Because the IEP is tailored to meet the needs of each student, it helps to ensure that he or she is placed in the least restrictive environment.

For my students, the setting that best meets their needs and limitations is their home. Many of my students could be described as “at risk.” Most are behind in their studies due to prolonged illness. Medication often makes concentration difficult. Some lack motivation. Schoolwork is often unimportant to a child suffering with an illness such as cancer. Therefore, I must find ways to stimulate my student's interest in and modify my instruction to fit the ability and physical condition of the student in order to bridge the gap between homebound and his/her successful return to the normal school setting. These problems are compounded by having to teach all content areas (English, Math, Science, Social Studies, etc.) in a limited amount of time. The minimum requirement according to the Texas Education Agency is four hours per week. I usually work with each of my students one hour per day four days per week. However, I do have some advantages. All of my instruction is done one-on-one. This provides each student the opportunity for individual participation and immediate feedback. Teaching strategies are developed for individual learning styles along with instructional modifications of the content areas in the IEPs. Community

BACKGROUND

Some students need to receive instruction in places other than the regular school setting. The Community Services Program offers just such options for eligible students up to age 21 who are confined to hospitals, at home, or at other special sites as a result of physical, mental, or emotional needs. Working closely with parents and local agencies, the staff of Community Services provides these students with continuous learning opportunities in the setting that best meets a given student’s needs and limitations.

Community Services has three components: Hospitals, Homebound, and Agencies. I teach in the Homebound Program. Community Services makes available regular and special instruction in the home to students who are expected to be out of school for four consecutive weeks or longer for health reasons, as documented by a physician. The continuation of studies ensures a smooth transition back to the regular classroom when the student is ready to return.

As a homebound teacher, my students come from various schools across the Houston Independent School District. Many of my students could be described as “at-risk.” They have academic deficits from prolonged illness. Many are on medication and some lack motivation. If you have cancer, your schoolwork may seem less important. Therefore, I must adapt my instruction for students working below grade level. I must modify my instruction to fit the ability and physical condition of the student to facilitate his/her successful return to “Home” school.

I am responsible for teaching students of various grade levels (sixth through twelfth). Also, I must teach all content areas (English, History, Science, Math, etc.) for each student. My problems are compounded by having a limited amount of time to spend with each student, usually one hour per day four days per week. Therefore, I am not able to spend as much time on each subject as I would in a normal classroom. However, I do have some advantages. All my instruction is done on a one-on-one basis. This provides each student with immediate feedback and numerous opportunities for individual participation. All homebound teachers have a Compaq LapTop Computer and access to the TI-82 or TI-83 Calculators.

Geometry Texas Essential Knowledge and Skills (TEKS)

111.34.b.2. **The student analyzes geometric relationships in order to make and verify conjectures.**

111.34.b.2.A. The student uses constructions to explore attributes of geometric figures and to make conjectures about geometric relationships.

111.34.b.2.B. The student makes and verifies conjectures about angles, lines, polygons, circles, and three-dimensional figures, choosing from a variety of approaches such as coordinate, transformational or axiomatic.

111.34.c. **The student identifies, analyzes, and describes patterns that emerge from two- and three-dimensional geometric figures.**

111.34.c.1.A. The student uses numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.

111.34.c.1.B. The student uses properties of transformations and their compositions to make connections between mathematics and the real world in applications such as tessellations or fractals.

111.34.d.2. **The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.**

111.34.d.2.A. The student uses one- and two-dimensional coordinate systems to represent points, lines, line segments, and figures.

111.34.f. **The student applies the concepts of similarity to justify properties of figures and solve problems.**

111.34.f.1.A. The student uses similarity properties and transformations to explore and justify conjectures about geometric figures.

111.34.f.1.D. The student describes the effect on perimeter, area, and volume when the length, width, or height of a three-dimensional solid is changed and applies this idea in solving problems.

111.34.e.1. **The student extends measurement concepts to find area, perimeter, and volume in problem situations.**

111.34.e.1.A. The student finds area of regular polygons and composite figures.

It is hoped that upon completion of this unit, the student will not only have gained a knowledge of the TEKS, but will also have had a learning experience that will motivate him to appreciate the study of geometry.

GEOMETRIC SOLIDS

When we initially look at a building, we see geometric solids. We see a myriad of rectangular prisms, the workhorses of architecture, along with triangular prisms, pyramids, cylinders, cones, and spheres. Sometime these solids are combined with each other to form complex structures. This relationship between geometric solids is especially noticeable in roof shapes. While the basic building shape may be a rectangular prism, the roof may be a triangular prism. It could consist of additional triangular prisms along with rectangular prisms, spheres or cones that may project beyond and/or above the roof.

My students will begin to explore shapes by first becoming familiar with geometric solids. I will use Relational GeoSolids to introduce the rectangular, square, triangular, and hexagonal prisms as well as the square and triangular pyramids along with cylinders, cones, sphere and hemispheres. A prism is a three-dimensional figure whose top and bottom faces are congruent polygons in parallel planes. Both the top and bottom faces can be referred to as a base. The side or lateral faces are all rectangular. Most solids are named according to the shape of their bases. A pyramid is a three-dimensional figure with a single base in the shape of a polygon and lateral faces that are all triangular. These triangular faces all meet at a single point (or vertex) called an apex located opposite the base (Lindebrekke, 14). A cylinder is a three-dimensional figure similar to a prism except its bases are congruent circles instead of polygons. Thus a cylinder does not have lateral faces (Lindebrekke, 50). A cone is a three-dimensional figure that is similar to a pyramid, except it has a circular base. Like a cylinder, a cone does not have any lateral faces (Lindebrekke, 56). A sphere is a three-dimensional set of points that are the same distance from a center point (Lindebrekke, 56). A hemisphere is one half of a sphere.

After the students become familiar with these basic shapes, I will use the architecture of buildings to reinforce and increase their awareness of geometric solids. In our walking tours of Houston buildings and neighborhoods we saw many examples of rectangular buildings. The El Paso Energy Building along with several buildings on the St. Thomas University campus serve as examples. The Astrodome is a cylinder while the Dynegy Building is a cylinder pulled apart. There is a suggestion of a pyramid in the entrance to Pennzoil Place while the Jesse H. Jones Building, Rothko Chapel and the University of Houston's Stephen Power Farish Hall embody the octagon.

While it is true that the architecture of buildings provides a perfect setting for the study of geometric solids, it is also true that most buildings do not consist of a single shape. They are considerably more complicated. They, in fact, consist of interdependent shapes of varying sizes and proportions that can be seen in Bank of America Center's combination of rectangles, triangles and barrel vaults. I want my students to look at a building and to be able to pick out the overriding or most basic geometric solid or solids that comprises their building.

Lesson Plan

Objective

The student will identify the geometric solids found in architecture.

Activities

1 The student will bring several pictures of buildings to class. At least one of the buildings is to be from the student's community. The student can get these pictures by (a) using a camera to take pictures of a building in his community, (b) cut out pictures from a local newspaper or magazines, or (c) print a picture of a building found on the internet.
2 The student will use a felt point pen to outline the basic shape of his buildings.
3 The student will mount his pictures on a bulletin board.

Materials Needed

Cameras, felt pens, magazines.

GEOMETRIC SOLIDS: NET PATTERNS

Several things should become apparent after discussing the students' pictures. First, can we really estimate what the geometric solid is? A solid is a three-dimensional figure and several of the pictures may contain only two dimensions. Here, we can help the student see the relationship between a geometric prism and a geometric shape. This should also help the student to understand the concept of volume.

Second, most of the buildings will not fit into a simple geometric solid. As mentioned before, most buildings consist of interdependent shapes of varying sizes. Buildings can be seen as objects or containers occupying or defining volumes of space. We can manipulate these buildings by either adding to or subtracting from this volume. For example, the Ezekiel W. Cullen Building and Houston City Hall look like they were constructed from a series of cubes. This again brings us to volume as buildings have a roof and walls surrounding a space used for living, manufacturing and other purposes.

The next exercise is for the students to determine how to construct geometric solid of their picture. In most cases this will be a rectangular prism although it might be a pyramid or cylinder. The student must determine how many faces (sides) the building has along with the top and bottom. He then must visualize how to unfold his solid to form a pattern. The student may have difficulty visualizing this. However, I believe it best to have the student explore this rather than just showing examples. The two-dimensional pattern that can be folded to create a representation of a three-dimensional solid is called net pattern.

Lesson Plan

Objective

The student will be able to sketch the various net patterns that form the geometric solids.

Activities

1 The student will be able to create a pattern from the basic geometric solid of his building on cardboard.
2 The student will cut out and fold his pattern to make sure it forms the shape of his geometric solid.
3 If the student's building is a cylinder, the student can first trace the circle from the top of the base and then use string to determine the circumference of the cylinder. Or they can use the formula for the circumference of a circle.

Materials Needed

Ruler, pencil, scissors, and cardboard.

GEOMETRIC SOLIDS: SURFACE AREA, LATERAL AREA AND VOLUME

Using architectural buildings as examples makes the concepts of lateral area, surface area, perimeter, square feet, and cubic feet easier to understand. If the building is a prism, it will have a polygon as a base (foundation) and another that is the same size and shape (congruent) as a top (roof). It will have the same number of faces (sides) as the base. All of the faces will be rectangles. The sides (walls) of these buildings comprise what is known as the lateral area of the prism in geometry. If we add the base and top, we have the total surface area.

In architecture the lateral area is extremely important because windows, doors, bookcases, entertainment centers, pictures, etc. are dependent on these surfaces. Also the length (perimeter) of the lateral faces is one of the major factors in determining the cost of construction. In general the greater the perimeter the greater the cost. The student will use the pattern he created from his building to determine its lateral and surface areas. The student will then expand on these relationships by using cubic blocks to represent various rectangular buildings. The number of cubic blocks used to construct the building will be its volume.

Lesson Plan

Objectives

1 The student will be able to describe the relationship that exists between surface area, lateral areas, perimeter, square feet and cubic feet.
2 The student will be able to determine surface area, lateral area, perimeter, square feet and cubic feet.

Activities

1 Each student will be given 1500 cubic blocks. Each block will represent one cubic foot.
2 The student will construct a rectangular building ten feet by eighteen feet and nine feet high out of the blocks.
3 The student will record the surface area, lateral area, perimeter, square feet and cubic feet (volume) for the construction.
4 The student will construct another building, this time in the shape of the letter L out of the same number of blocks and repeat Step # 3.
5 The student will compare the results in Step #3 to Step # 4. What has changed? What has remained the same?
6 The student will construct a representative two-story building. Eighteen feet high out of the same number of blocks and repeat Step # 3.
7 The student will compare the results to Step #3. What has changed? What has remained the same?

Materials Needed

1500 cubic blocks, square paper, pencil.

SHAPES AND PATTERNS

Geometry consists of three types of properties: size, shape, and position. Often, we compare objects by placing one against the other to see if they match. Questions about size and shape can be answered by movement.

There are three basic ways to move a figure. Translation slides a figure along a line without twisting it. We saw this in the row houses. The plan for the house was laid out, the house constructed and then the plan was translated (slid) horizontally to the right or left a certain distance and another house constructed. This process was repeated several times with the end result being identical houses lined up along the street. This creates a kind of balance and can be referred to as translation symmetry. Most subdivisions are built this way. However, the subdivision might use three or four different plans to create variety. Another example of this type of symmetry can also be seen on most building elevations in which identical windows line up as in most of the downtown office buildings. Translation of an object leads to movement in a coordinated plane. A coordinate plane connects Geometry to Algebra. Downtown Houston is laid out like a coordinate system. You could draw a horizontal line along Walker (X-axis) and a vertical line along Main Street (Y-axis). The origin of this grid would be the intersection of Walker and Main.

Lesson Plan

Objective

The student will use a coordinate system to represent points and figures.

Activities

1 The student will create a coordinate grid on a map of downtown Houston by (a) using a felt tip marker to draw an X-axis along Walker Street and (b) draw a Y-axis along Main Street.
2 Beginning at the intersection of Walker and Main (origin), what building would be two blocks south and four blocks north if you were facing east?
3 Describe how you would have to walk to get to the Reliant Energy Plaza.
4 How far would you have to walk to go from the Foley's Building to Two Shell Plaza? Could you take a different route?

Materials Needed

Map of Downtown Houston, felt tip pen, pencil, paper.

SHAPES AND PATTERNS; SYMMETRY AND GRID SYSTEMS

Another method used by subdivisions to create variety is to flip the plan over. This type of movement is called reflection and is often referred to as the mirror image. It is the image you would get if you were able to place a mirror on the reflection line and compare the mirror image to its reflection image. Apartments are often designed with the apartment on one side being the reflection of the apartment on the other side of the common wall. Reflection leads us to another type of symmetry. This is the most basic form of symmetry and can be referred to as bilateral symmetry. Bilateral symmetry is defined as the balanced distribution and arrangement of equivalent forms and spaces on opposite sides of a dividing line on a plane, or about a center or axis. It is the most prevailing form of symmetry found in architecture and the Architecture Building on the University of Houston campus where our class met is a classic example. A third, less common way to move a figure is rotation. This rotation can be about the origin or about a point. The houses along a cul-de-sac are examples of rotation around a point. Walter

Netsch's "Field Theory" and some houses designed by Frank Lloyd Wright are examples of rotations about the origin. (See Appendix: Walter Netsch and Frank Lloyd Wright.)

Shapes and patterns show up in architecture in two ways. First, they are seen in the façades of buildings. Windows, doors, columns and other structural elements are often arranged symmetrically. We saw this in the façade of the original Farm Credit Bank building, an example of reflection. Almost all building façades exhibit some form of symmetry. I would now have my students go back to their pictures and with a felt point pen outline and name the various shapes found in the façades of their building.

Symmetry, which dates back to the ancient Greeks, has always played an important role in architectural design. Again, modern mathematics has further generalized and formalized the concept of geometric symmetry, grounding it upon the idea of a group of geometric transformations (Mitchell, 30). These symmetry operations (translations, rotations, reflections) can be seen in most buildings. I would now have my students see if they could identify a form of symmetry found in the façades of their building.

Second, we see shapes and patterns in various grid and module systems and no shape plays as important a part in architecture as the rectangle. It is the most adaptable shape for human needs in all of geometry. Street grids, city blocks and lots (as we saw in downtown Houston), fields, rooms, doors, windows, and furnishings are usually rectangular. Most building materials are rectangular. Most structural systems are rectangular. The El Paso Energy Building is an example of a rectilinear/structural system. You can sense this in many of the older buildings where the structure became part of the design. This structure was hinted at in One Shell Plaza. However, many new buildings do not reveal their structure. But, instead conceal it by covering with a skin. Within this rectilinear framework, almost everything needed for utility, comfort, and convenience can be integrated into a marvelously rational and harmonious system. Characteristically, the constructed environment is rectangular. It could be nothing else (Blackwell, 70). We saw this continuously in our field trips. The courtyards of the University of St. Thomas, the University of Houston and Rice University are a few of the examples of this rectangular environment.

Modern architecture for the most part has declared that form must follow function. However, some architects have tried to accommodate the architecture to the geometry. Charles G. Woods is one of the architects that put the geometry of architecture first. Almost by definition, if you are working with pure forms (squares, rectangles, triangles), you must design somewhat from the outside in (Woods, 38). It was logical that the first module systems of design would involve the rectangle. Charles Woods in his book *A Natural System of House Designs* says that all residential design can be based on a 4'-0" square module. All of his house designs are based on this module. Everything is on the module – columns, doors, windows, closets, chimneys, sinks, and tubs (Woods, 18). Grids can save money by avoiding waste. Many building materials, plywood, concrete block, joist and rafter spacing, etc. work with the 4'-0" square module. As mentioned before we have a natural tendency to search for order and the module grid brings a consistency and order to buildings. Like the painter Mondrean you can create seemingly endless variations on a simple and uniform system (Woods, 4).

Modules have been used by many architects throughout history. Frank Lloyd Wright carried the use of module design to a higher level incorporating polygonal and angular module systems to his designs. Wright based his designs for Hanna House and the Audrey House on variations of hexagonal geometry. He based the Sundt House and the Hotel Part of San Marcos in the Desert on the complexities of the triangular module. Except for the Chapel, the St. Thomas University Courtyard and the buildings connected to it could have been based on a rectangular module.

Walter Netsch has been able to combine several different geometric shapes into one grid called "Field Theory." This combination of patterns results in a continuous proportional system with infinite mathematical variations. As in contemporary painting, sculpture and music, continuity, ambiguity, overlapping, scale change, pattern and shape are constant ordering elements (Schmertz, 116). This results in a variety of forms and sizes from the small and intimate to the large similar to the patterns underlying the geometrical art of the Arabs. This theory evolved from Netsch's reactions against the formal purity of the rectangular grid. His designs for the Chicago Circle Campus of the University of Illinois are based on identical squares rotated around their centers. These squares were then translated so that they would touch at the corners in a variety of ways. (See Appendix: Walter Netsch.) Duncan Hall at the Rice University could be an example of a building accommodated to geometry.

Netsch has continued to invent new fields. Netsch has also experimented with an architecture of growth. He applied the theories of fractal geometry to architecture. Fractal geometry is the study of mathematical shapes that display a cascade of never-ending, self-similar, meandering detail as one observes them more closely. The fractal dimension is a mathematical measure of the degree of meandering of the texture displayed. Natural shapes and rhythms display this progression of self-similar form. Examples of these include leaves, three branching, mountain ridges, flood levels of a river, wave patterns and nerve impulses. Fractal concepts are being used in many fields from physics to musical composition. Architecture and design, also concerned with the control or rhythm, can benefit from the use of this relatively new mathematical tool (Bovill, 1). Netsch used this in his design for the new Miami University Art Museum. The building is based on squares of increasingly larger size, in which the proportional relationship between them is obviously determined by geometry.

Mitchell in his book *The Logic of Architecture* explores growth patterns by trying to develop a universal language of design. He is working toward a system of rules that would manipulate identical units of shape by addition to create architecture. Rules of grammar have been developed for the half-hexagon along with squares and rectangles. Relationships created by overlapping identical shapes are also explored.

Lesson Plan

Objectives

1 The student will develop spatial perception and visualization skills.
2 The student will understand the concept of area by using nonstandard units of area.

Activity

The student will design a floor plan for a house that includes a family room, dining room, kitchen, three bedrooms, two and one-half bathrooms, and a garage by (a) developing a nonrectangular grid system or (b) using a nonrectangular shape as the basic module. (See Appendix: Nonrectangular Grid Systems.)

Materials Needed

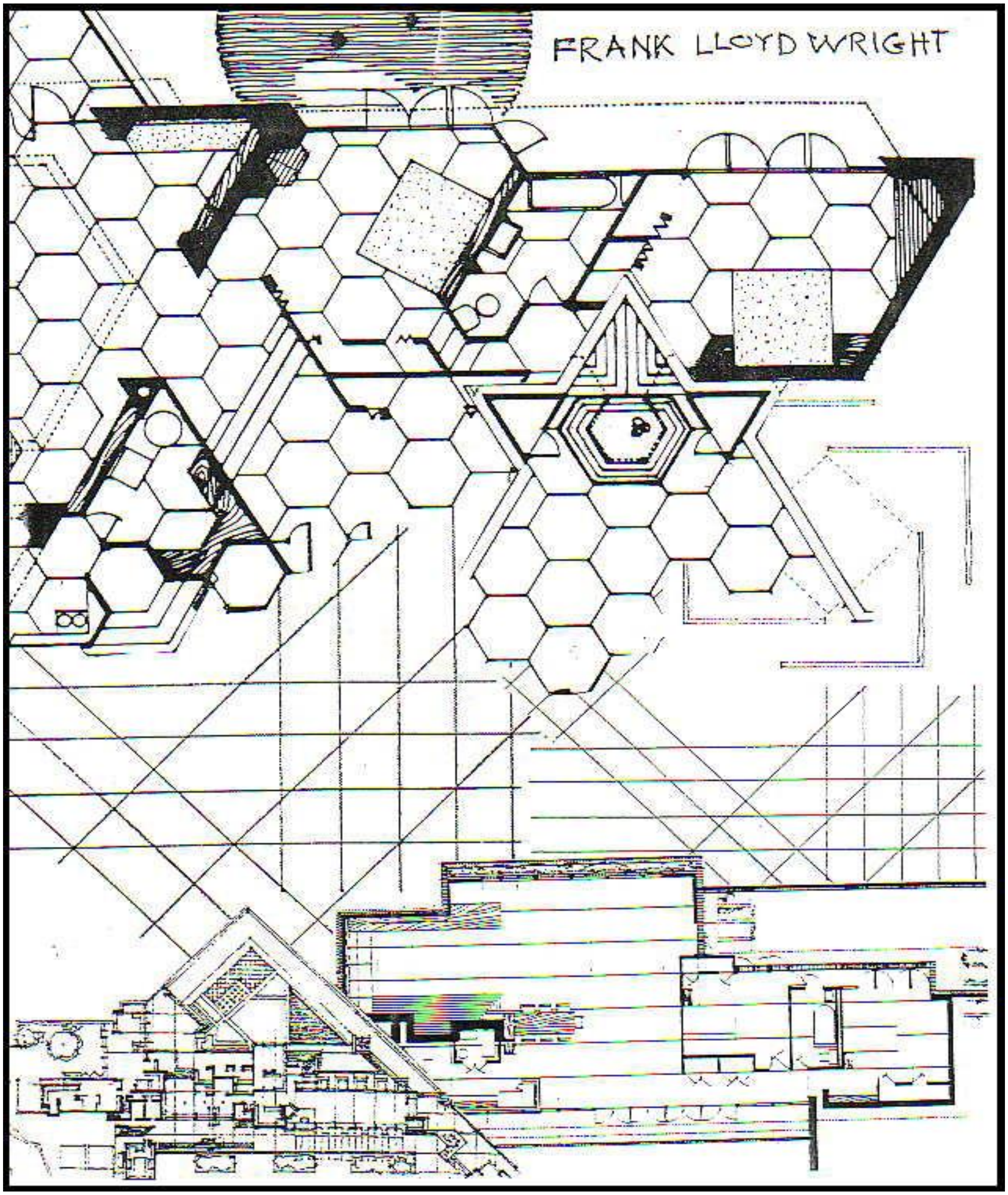
Pattern blocks, grid paper, dot paper, pencils, scale.

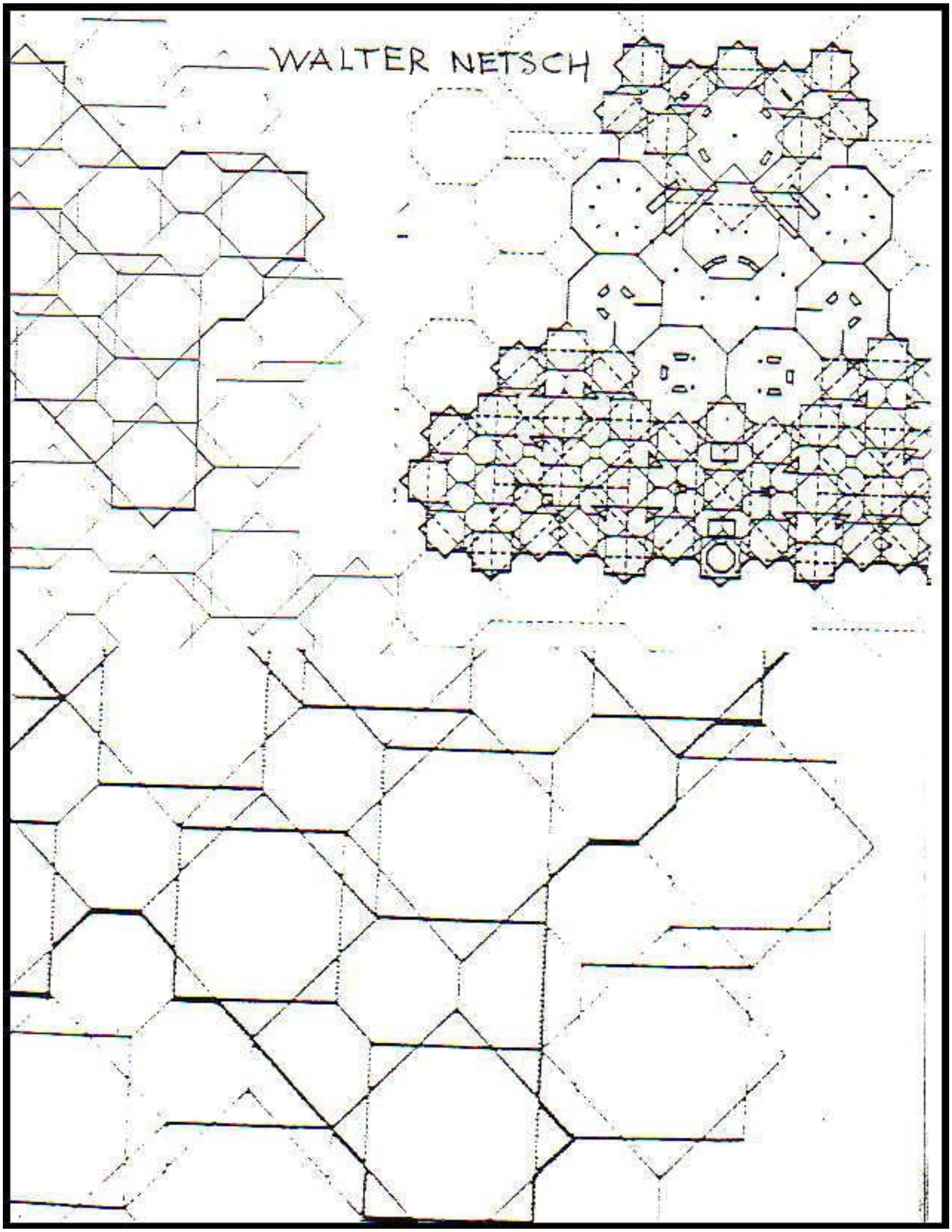
CONCLUSION

I hope I have provided a basic framework to help students make connections between geometry and architecture. I believe that using real world examples help motivate students and pique their interest in a such way that after this unit students will see their environment differently. Their world will not be the same.

While I am hopeful that I have given you a few ideas you could use in your classroom, this unit has only scratched the surface. I am sure you will come up with many more lessons exploring the connections between Geometry and Architecture. Good Luck!

APPENDIX

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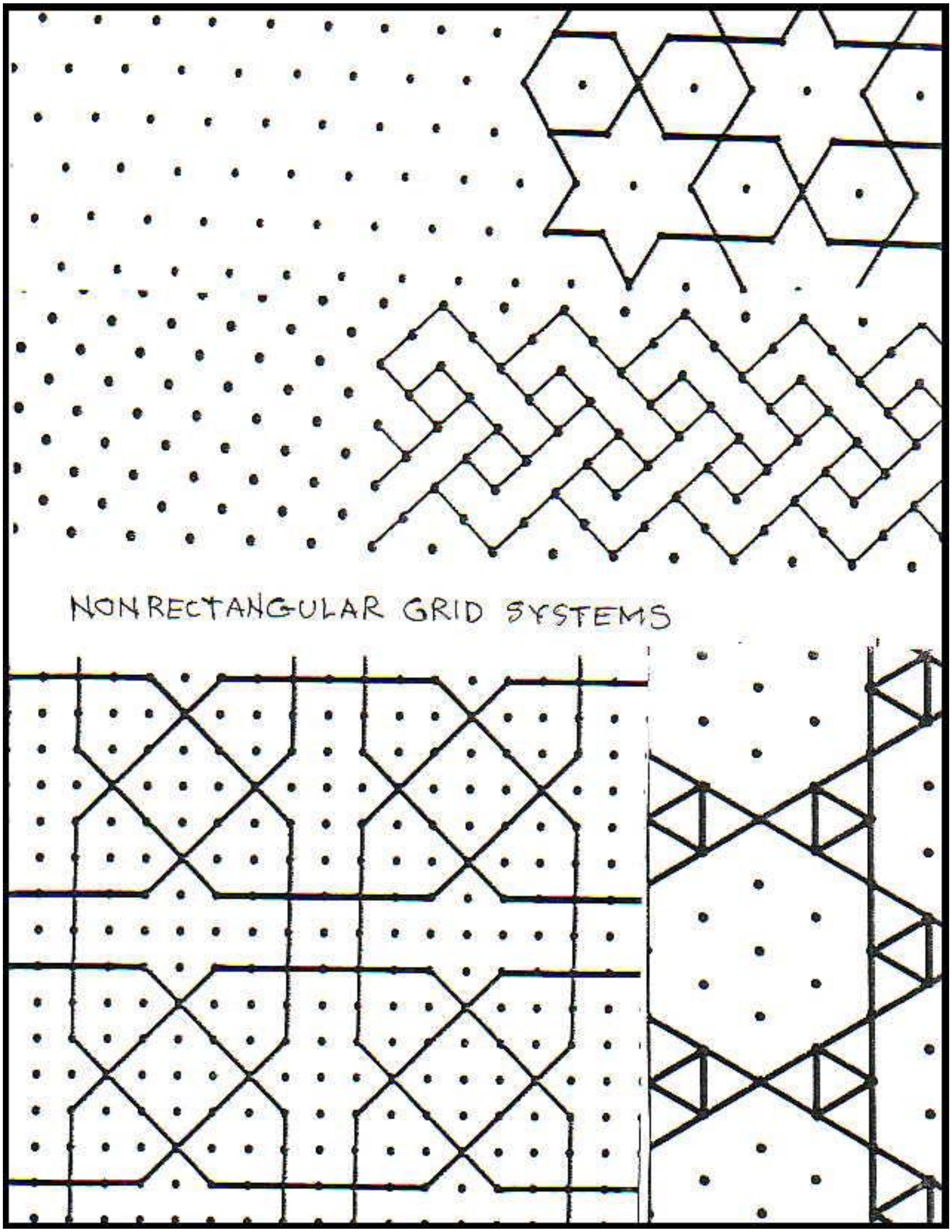
NONRECTANGULAR GRID SYSTEMS

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This book provides the layman with an understanding of the basic geometric principles. It discusses all of the basic geometric shapes found in architecture.

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Bovill, Carl. "Fractal Geometry in Architecture."
A discussion of fractal geometry in architecture.

Equals at the Lawrence Hall of Science. *Growth Patterns*. Berkeley, Calif.: University of California, 1994.
This is a book that was developed for middle-school mathematics focusing on linear and exponential growth functions.

Fox, Stephen. *Houston Architectural Guide*. Americans Institute of Architects. Houston: Houston Chapter and Herring Press, 1999.
A detailed guide, with pictures and comments, of buildings in Houston.

Lindebrekke, Karen and Dana Hupert. *Investigating with Relational GeoSolids*. Vernon Hills, Ill.: ETA, 1999.
This is a fun and interesting workbook created to provide many hands-on mathematical investigations into geometry. It is an excellent book for teachers that are not familiar with geometry.

Mitchell, William J. *The Logic of Architecture: Design, Computation, and Cognition*. Cambridge, Mass.: MIT Press, 1990.
This is a very technical and complicated book on the theory of design. This book is written for architects and architecture students concerned with the design process.

Nereim, Anders. "Walter Netsch: Having a Field Day." *Inland Architect* 34 (1990): 60-67.
This article discusses the history of Netsch's Field Theory and its use in the design of libraries at Northwestern University and the University of Chicago.

Schmertz, Mildred F. "A New Museum by Walter Netsch of SOM Given Order by His Field Theory." *Architectural Record* 167 (1980): 111-20.
This article discusses the design of the Miami University Art Museum based on Netsch's Field Theory and proportional design.

Woods, Charles G. *A Natural System of House Design*. New York: McGraw-Hill, 1996.
This book is about how one architect, Charles G. Woods, developed a module system for designing homes. It includes his philosophies along with a step-by-step procedure for the design of residential structures. It provides many examples of his houses.