

Probability in Everyday Life

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INTRODUCTION

Certain students need to receive special instructional services in places other than the regular school setting. The Community Services Program offers just such options for eligible students up to age 21 who are confined to hospitals, at home, or at other special sites as a result of their physical, mental, or emotional needs. Working closely with parents and local agencies, the staff of Community Services provides these students with continuous learning opportunities in the setting that best meets a given student's needs and limitations.

For HISD students who are expected to be out of school for four consecutive weeks or longer for health reasons, as documented by a physician, Community Services makes available regular and special education instruction in the home. The continuation of studies ensures a smooth transition back to the regular classroom when the student is ready to return.

As Homebound teachers, our students come from various schools across the Houston Independent School District. Many of our students could be described as "at-risk". They have academic deficits from prolonged illness. Many are on medication and some lack motivation. If you have cancer, your school work may seem less important. Therefore, we must adapt our instruction for students working below grade level. We must modify our instruction to fit the ability and physical condition of the student to facilitate his/her successful return to "Home" school.

We are responsible for teaching students of various grade levels (6-12). Also, we must teach all content areas (English, History, Science, Math, etc.) for each student. All our instruction is done on a one on one basis. This provides each student with immediate feedback and numerous opportunities for individual participation.

The study of probability and statistics is emphasized for 8th graders. This curriculum unit will introduce students to the 8th grade TEKS for probability and statistics.

(8.11) Probability and statistics. The student applies concepts of theoretical and experimental probability to make predictions. The student is expected to:

- (A) find the probabilities of compound events (dependent and independent);
- (B) use theoretical probabilities and experimental results to make predictions and decisions;
- (C) select and use different models to simulate an event.

(8.12) Probability and statistics. The student uses statistical procedures to describe data.
The student is expected to:

- (A) select the appropriate measure of central tendency to describe a set of data for a particular purpose;
- (B) draw conclusions and make predictions by analyzing trends in scatter plots; and
- (C) construct circle graphs, bar graphs, and histograms, with and without technology.

(8.13) Probability and statistics. The students evaluates predictions and conclusions based on statistical data. The student is expected to:

- (A) evaluate methods of sampling to determine validity of an inference made from a set of data; and
- (B) recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.

(TEXAS EDUCATION AGENCY - WEB-SITE)

The unit will be designed for use in 8th grade Math classes, Algebra classrooms, and TAAS tutorials. We hope to present the study of probability and statistics in an entertaining way. The module will contain a wide variety of strategies and activities. Upon completion of the unit, students will not only have gained knowledge of the content areas, but also a learning experience that will motivate them to appreciate the study of mathematics and feel that they, too, can beat the odds.

BACKGROUND

Probability and statistics is often referred to as if it is a single entity. The Houston Independent School District links them together. However, while often interrelated, they are two separate fields of study. This enables us to integrate the topics of probability, statistics, graphing and measurement together rather than dealing with each separately. It is believed that integrating these topics helps maximize students learning. However, for this paper I will deal only with the section on probability.

The study of probability theory originated in the 17th century. Louis XIV of France was in power and he wanted his Nobles to stay out of affairs of state. However, while reducing their political power, he left them with wealth and leisure time. They used this time to play various games. It was during this time mathematicians began calculating the odds in various games of chance, such as dice, cards and roulette. Probability is now used to help us understand science insurance, politics, sports and other aspects of our life (Singh, 216).

How do we begin the study of a new subject or area of study? Every subject whether economics, biology or probability theory, wants to be unique. It wants to stand out and be

important. How does it achieve this goal of separation from other subjects? It creates a vocabulary that we must understand in order to converse intelligently about the subject.

Where does this vocabulary originate? It can come from two sources. First, there are those words we borrow from every day speech and bestow on them a distinct meaning. Second, are words made up just for the subject and have no other meaning (Singh, 216)?

In every day conversation we use the word probability to express the strength of our beliefs. We speak of the probability of tossing a “Head” when we throw a coin or of picking an “Ace” from a deck of cards. We also talk of the probability of the Houston Astros winning the world series and the probability of the defendant being innocent. Although these examples sound the same, they are different.

When we throw a coin or pick a card, we can repeat that event as many times as we wish. After we have repeated the event a large number of times, we can come up with a frequency ratio. A frequency ratio is a ratio of the number of times an event actually occurs to the total number of times the event was done. This is called experimental probability. Probabilities found by counting and classifying all possible outcomes is called the theoretical or statistical probabilities. This is representative of the first group.

When we talk about the Astros winning the World Series, we have a non-repetitive event. We have to estimate the degree of belief we have in the statement. This is done on an intuitive level and is called inductive probability. For now we will confine our study to theoretical probability.

Student Objective

The student uses theoretical probabilities and experimental results to make predictions and decisions. I will begin with an assignment on coin-tossing to illustrate the difference between theoretical and experimental probability. I chose coin-tossing because of its simplicity.

It is easy for the students to grasp and I believe their predictions will be close to reality. I will follow this with the Monte Hall Problem. I believe when we compare the students’ predictions to the experimental results with the Monte Hall Problem, most will have guessed incorrectly. The Monte Hall Problem will help them to understand why probability theory is full of misunderstanding and paradox. Why, at times it can be frustrating yet rewarding.

To repeat, students have a hard time understanding the concepts of probability theory. We do not know what strategies are best suited for adolescents. However, it is believed that a student’s lack of experience with physical situations that embody probability concepts seems to be an important part of the explanation of poor performance. Therefore, a hands-on strategy seems to be preferable.

The outcome of an event can be expressed in two ways. If an event can succeed in (S) ways and fail in (F) ways, then the probabilities of success and the probabilities of failure are as follows:

$$P(s) = \frac{S}{S+F} = \frac{\# \text{ Successes}}{\text{Total \# of Outcomes}}$$

$$p(f) = \frac{F}{S+F} = \frac{\# \text{ Failures}}{\text{Total \# of Outcomes}} \quad (\text{BOYD} - 732)$$

If an event cannot fail, it has a probability of 1 and if it cannot succeed, it has a probability of 0. Therefore, the probability of any event will always be between 1 and 0, and expressed as a fraction.

Finally, probability is not to be confused with odds. The odds of an event can also be expressed in two ways; the odds in favor = the number of successes: number of failures or the odds against = number of failures: number of successes. For example the probability of picking a queen from a deck of cards would be 1/13 while the odds of picking a queen would be 1:12 and the odds against picking a queen would be 12:1.

The student will find the probabilities of compound events (dependent and independent). After these initial concepts are comprehended, the rest of the study of probability is concerned with the development of formulas to calculate complex events and to combine events. These formulas are based on two of the fundamental rules of math. These are the rules of addition and multiplication. However, students have a great deal of difficulty in distinguishing between the additive and multiplicative situations. Also, in order to use these formulas, the student must be able to employ the fundamental counting principle, permutations and combinations.

Often the fundamental counting principle, permutations and combinations are taught before the study of probability. It is felt that formulas should be a product of exploration and discovery. Therefore, tree diagrams are often used to model the fundamental counting principle. The definition of the fundamental counting principle is that if Event M can occur in M Ways and is followed by an Independent Event N that can occur in N Ways, then the Event M followed by the Event N can occur in MxN Ways. For example, if you have 4 shirts, 3 pairs of pants, and 2 pairs of shoes, how many different combinations of attire can you wear? There would be 4x3x2 or 24 combinations.

When a group of things are to be arranged in a certain order, you have a permutation. In a permutation, order is important. For example, if you want to know the number of ways to arrange 10 things taken 3 at a time it would be written as follows P(10,3) or 10P3 and is read the permutation of 10 objects taken 3 at a time and the definition of permutation P(n,r) or nPr is the number of permutations of n objects taken r at a time is defined as follows:

$$P(n,r) = \frac{n!}{(n-r)!} \quad (\text{Boyd, 718})$$

If order is not important, for example, to win the Lottery, you need to have 6 numbers. It does not matter in which order these numbers are chosen. An arrangement or listing in which order is not important is called a combination. It is written $C(n,r)$ or nCr and is defined as the number of combinations of n distinct objects taken r at a time or

$$C(n,r) = \frac{n!}{(n-r)!r!} \quad (\text{Boyd, 726})$$

Having quickly reviewed the fundamental counting principles, permutations and combinations, we can now get back to our study of probability theory using our additive and multiplicative formula.

If two events, A and B are independent then the probability of both events occurring is found as follows $P(A \text{ and } B) = P(A) \times P(B)$. If two events A and B are dependent, then the probability of both events occurring is found as follows: $P(A \text{ and } B) = P(A) \times P(B \text{ following } A)$. For example, suppose we have a bag with 5 red balls and 4 green balls and you want to calculate the probability of picking 2 red balls in two draws in the first case the ball is replaced after the draw. The probability for the first draw and the second draw would be $5/9$ and the total probability would $5/9 \cdot 5/9 = 25/81$. In the second case the ball is not replaced. The total probability now becomes $5/9 \cdot 4/8 = 20/72$ or $5/18$ for picking two red balls.

Note that you must decrease both the number of red balls and the total number of balls. Students have a problem understanding this part/whole relationship and often will just reduce the number of red balls leaving the sample space as is. However, in both examples you must use multiplication.

Again if you have two events, you must understand how they are related. Sometimes two or more events are such that not more than one of them can occur in a single trial. For example, in tossing a single die, what would be the probability of “throwing a four” or “throwing a seven?” Since a die cannot be both a four and a seven at the same time, the events are said to be mutually exclusive. If two events, A and B, are mutually exclusive, the probability that either A or B occurs is the sum of their probabilities. $P(A \text{ or } B) = P(A) + P(B)$ in the above example the probability of “throwing a four” would be $1/6$ and the probability of “throwing a seven” would be $1/6$. Therefore, the probability of “throwing a four or seven” would be $1/6 + 1/6 = 2/6$ or $1/3$. What would be the probability of drawing a king and spade from a deck of cards? In this case, it is possible to draw a card that is both a king and a spade. This is called an inclusive event. This concept can be modeled by the use of a ven diagram. It would show the overlapping area. In this case the king of spades which is both a king and a spade. To prevent us from counting this card twice, we must subtract the probability (king of spades) from the sum. Therefore, the

probability of an inclusive event is as follows: If two events, A and B, are inclusive, then the probability that either A or B occurs is the sum of their probabilities decreased by the probability of both occurring $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. In the above example, that would be the probability of a kings $4/52 +$ spade $13/52 -$ the king of spades $1/52 = 4/13$ which is the probability of drawing a king or a spade.

Conclusion

Probability plays a larger part in most people's lives than they realize. It is important that our students gain some real hands-on experience in understanding the mathematical concepts behind probability theory to enable them to avoid the confusion and fallacies associated with it.

For example, what would you do if you were on this jury?

Let us suppose a man living in England is accused of a murder or a rape. The only evidence against him is a sample of bodily material left at the scene of the crime. If the forensic scientist tells the jury that this sample could have been left by only one person in a million, it is quite possible the defendant will be convicted on this evidence, for the jury is likely to conclude that the probability of him being innocent is only one in a million. In fact it is nothing of the sort. If we suppose a total adult male population of twenty million, then we can expect about twenty men to match the forensic sample. Only one of these men is guilty and the other nineteen are innocent, so the probability of the suspect being innocent, if no other evidence is available, is $19/20$. The jury is not being asked to decide whether the suspect matches the sample; this piece of information is given.

They are required to decide if he, among those who do match, is the one responsible for the crime. They mistook both the sample space and the subset of outcomes required. (PARR - 8)

LESSON PLANS

Coin Tossing

Materials Needed:

1. Give students a coin
2. Give students paper to record tosses

I. Experimental Probabilities

A. Ten Tosses

1. Predict how many times the coin will come up heads in ten tosses.

2. How many times did the coin come up heads? _____
3. You can find the experimental probability of tossing a head by placing the number of heads over the total numbers of tosses.

$$\frac{\text{Number of Events Resulting in Heads}}{\text{Total Number of Events}} = \frac{\text{_____}}{10}$$

4. What is your experimental probability from tossing a coin ten times? Probability is recorded as a fraction. You must reduce fraction to lowest terms. _____

B. Fifty Tosses

1. Predict how many times the coin will come up heads in fifty tosses.

2. You can use your results from ten tosses to make a prediction by multiplying your answer from question 4 by fifty.

_____ X 50 = _____
3. You can find the experimental probability of tossing a head by placing the number of heads over the total number of tosses.

$$\frac{\text{Number of Events Resulting in Heads}}{\text{Total Number of Events}} = \frac{\text{_____}}{\text{_____}}$$

4. What is your experimental probability from tossing a coin fifty times? Probability is recorded as a fraction. You must reduce fraction to the lowest terms. _____

One Hundred Tosses

1. Predict how many times the coin will come up heads in one hundred tosses. _____
2. You can use your results from either your results in A or B by multiplying your answer from question 4 by one hundred.

$$\text{_____} \times 100 = \text{_____}$$

3. You can find the experimental probability of tossing a head by placing the number of heads over the total number of tosses.

$$\frac{\text{Number of Events Resulting in Heads}}{\text{Total Number of Events}} = \text{_____}$$

4. What is your experimental probability from tossing a coin one hundred times? You must reduce fraction to lowest terms. _____

II. Theoretical Probabilities

1. How many ways can you get a head if you toss a coin? _____
2. How many possible outcomes are there when you toss a coin one time?

$$\text{_____} \text{ Theoretical Probability} = \frac{\text{Ways of Getting a Head}}{\text{Total Number of Outcomes}}$$

4. To find out the theoretical probability for ten tosses, you can multiply this by ten.

$$\text{_____} \times 10 = \text{_____}$$

5. For fifty tosses you can multiply this by fifty.

$$\text{_____} \times 50 = \text{_____}$$

6. For one hundred tosses you can multiply by one hundred.

$$\text{_____} \times 100 = \text{_____}$$

7. Compare your answers for ten, fifty and one hundred tosses using theoretical probability to your answers using experimental probability.
8. What conclusions can you draw?

The Monte Hall Problem:

Boyd E. Hemphill of The John Cooper School and Dennis Donovan of the Galveston Bay Project Presented at The 10th Anniversary Celebration of The Rice University School Math Project

Description of Problem

The infamous probabilistic conundrum that has come to be known as “The Monte Hall Problem” has its history in the game show, Let’s Make a Deal. Here is the infamous Monte Hall Problem, as it appeared in Parade Magazine (September 1990):

Suppose you’re on a game show, and you’re given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say number 1, and the host, who knows what’s behind the doors, opens another door, say number 3, which has a goat. He then says to you, “Do you want to pick door number 2?” Is it to your advantage to switch your choice?

Question #1: Does it matter if you change your mind?

Question #2: What is the probability of winning if you remain with door number one?

Question #3: What is the probability of winning if you change your mind to door number two?

Surprisingly, the answer to question #1 is yes, it does matter. Intuitively, one would believe they have a one in two chance of winning, but this is not the case.

The answer to question #2 is that your probability remains one out of three for winning if you choose to remain with door number one. This implies the answer to question # 3 is two out of three because the probabilities must total one.

Frankly, I did not believe this result when it was explained to me at the Park City Math Institute in the summer of 1996. The result of my disbelief is the programs for the TI-85 Graphing Calculator below. In short, the program Montesim will allow you to act as the player and remain with your first choice or change your mind. The program “Monte” will allow you to make this decision up front and then run a specified number of trials based on that decision. In each case, enough trials will confirm the above results!

I intend to use Montesim first and allow the participants to come to their own understanding of the unexpected result. I will compile the results, and then use Monte to strengthen them. Finally, I will explain the reasons behind the result. I expect this to take about 10 minutes. The codes, and more extensive descriptions of the programs, follow below. Finally, I have listed some of the bookmarks I collected to help me put together

the programs and presentation. I hope you enjoy confirming this very interesting result, and I hope you have fun pondering its generalizations.

Boyd E. Hemphill

Game Program (Montesim)

The Program “Montesim” is an actual simulation of the game show “Let’s Make a Deal.” It allows you to choose your door, then it opens the wrong one “accidentally.” You are then asked whether you want to change your mind. After you decide, you are told whether you win or lose. The program keeps track of how many times you win or lose in each of the case- -change your mind or remain with your original choice.

Throughout the code below, I have placed comments that can be recognized by the change of font. These are intended to give you an idea of what the program is doing and suggestions for improvements you might make.

One improvement you may make right away is in error traps. I have made no provisions for them. An error trap is a logical test that would, for example, keep the user from choosing door number 4 or 2.3. This would corrupt the results of the simulation. In the case where an error trap may be useful, I will call for one and define its purpose.

Finally, Dennis and I worked for several days getting the program correct. We implore you to include our names and that of the Rice University School Math Project in the form specified in the program. If you decide to use our program at an in-service, please credit us in your handout. If you have the time, drop us an email message and let us know where and how you are using the program. We appreciate your time, and our bosses will appreciate us!

Set variable values for use in the program.

:8 door is randomly generated. The following If ... Then tests determine which wrong door to open.

```
:int 3rand+1->W
:If (D=1) and W=2)
:Then
:Disp “DOOR 3=ZONK”
:Goto T
:End
:If (D=1 and W=3)
:Then
:Disp “DOOR 2=ZONK”
:Goto T
```

```

:End
:If (D=2 and W=1)
:Then
:Disp "DOOR 3=ZONK"
:Goto T
:End
:If (D=2 and W=3)
:Then
:Disp "DOOR 1=ZONK"
:Goto T
:End
:If (D=3 and W=1)
:Then
:Disp "DOOR 2=ZONK"
:Goto T
:End
:If (D=3 and W=2)
:Then
:Disp "DOOR 1=ZONK"
:Goto T
:End
:If (W=1 and D=W)
:Then
:Disp "DOOR 2=ZONK"
:Goto T
:End
:If (W=2 and D=W)
:Then
:Disp "DOOR 1=ZONK"
:Goto T
:End
:If (W=3 and D=W)
:Then
:Disp "DOOR 2=ZONK"
:Goto T
:End
->y:9->n
:0->B:0->D:0->E:0->N:0->U:0->V:0->W:0->Y:0->Z:0->I:0->J
:ClrHome

```

The introduction. This only happens the first time you start the program.

```

:Disp ""
:Disp "LETS MAKE A DEAL"

```

```

:Disp “ “
:Disp “ B. HEMPHILL”
:Disp “ D. DONOVAN”
:Disp “ [ENTER]”
:Pause
:ClrHome
:Disp “ WITH“
:Disp “ “
:Disp “ THE RICE”
:Disp “ UNIVERSITY”
:Disp “ SCHOOL MATH”
:Disp “ PROJECT”
:Disp “ “
:Disp “ [ENTER]”
:Pause
:Lbl A

```

The program asks for a door. This is the first place an error trap could be used to force the user to pick 1, 2, or 3.

```

:ClrHome
:Disp “ CHOOSE DOOR”
:Input “ (1, 2, 3)”, D

```

The next line is how the winning
A message indicating “Monte’s Mistake” is given and the user is asked to decide whether to change their mind or remain with their first door. Again, nothing in the program keeps them from picking the door already open. An error trap to force the user in to such a decision would be good here. The problem is that the value of the mistakenly open door is never assigned above. You will have to rectify this.

```

:Lbl T
:Disp “ “
:Disp “ OOPS”
:Disp “ “
:Disp “CHOOSE AGAIN”
:Disp “FROM REMAINING”
:Input “DOORS”, E

```

The program determines if you have changed your mind so it can track the appropriate statistics.

```

:If D=E
:Then

```

```
:Goto R
:Else
:Goto S
:End
```

The program determines if you win or lose and notifies you. The two similar If ... Then statements are for the two different situations- -remain with first choice or change mind.

```
:Lbl R
:If E=W
: Then
:Disp "YOU WIN"
:I+1->I
:Else
:Disp "ZONKED AGAIN"
:J+1->J
:End
:Goto Z
:Lbl S
:If E=W
:Then
: Disp "YOU WIN"
:U+1->U
:Else
:Disp "ZONKED AGAIN"
:V+1->V
:End
```

Stats are given in table form, and a choice to run the program again or exit.

```
:Lbl Z
:ClrHome
:Output (1, 6, "STATS")
:Output (2, 1, "WINS")
:Output (2, 8, "LOSSES")
:Output (3, 2, "CHANGED MIND")
:Output (4, 1, U)
:Output (4, 8, V)
:Output (5, 1, "UNCHANGED MIND)
:Output (6, 1, 1)
:Output (6, 8, J)
:Output (7, 1, "[ENTER] CONTINUE")
:Output (8, 3, "[ON] BREAK")
```

```
:Pause  
:Goto A
```

Rapid Simulation (Monte)

The program Monte is a vast modification of a short program written by John Powers. It assumes “Monte’s Mistake” has already occurred and allows you to choose in advance if you would like to change your mind or remain with the first door chosen. Then it asks you for the number of such trials to work with. If you have the time, you could run 1,000,000 but this would take hours. The results given are the number of wins and losses you would experience for the number of trials you pick. This is a fast way to demonstrate the interesting answer to the problem.

Again, I will make short comments in the code to help you determine what the program is doing. I could find no need for error traps except the one I included in the beginning.

Again, I ask that you include Mr. Powers’ name and my own in the program in the specified form. If you decide to use our program at an in-service, please credit us in your handout. If you have the time, drop me an email message and let me know where and how you are using the program. I appreciate your time, and my boss will appreciate me!

The introduction. This only happens the first time you start the program.

```
:ClrHome  
:Disp “ Monte”  
:Disp “ “  
:Disp “ BY”  
:Disp “ JOHN P. POWERS”  
:Disp “ “  
:Disp “ MODIFIED BY”  
:Disp “BOYD E HEMPHILL”  
:Pause
```

The program asks you to which way you would choose to go- -remain with the same door or change your mind.

```
:Lbl K  
:ClrHome  
:Disp “ALWAYS CHANGE”  
:Disp “MIND PRESS 8”  
:Disp “ “  
:Disp “ALWAYS STAY WITH”  
:Disp “FIRST CHOICE”
```

```
:Disp "PRESS 9"  
:Input M
```

The following is an error trap that forces the user to choose 8 or 9. (Note: <> means not equal on test menu)

```
:If (M<>8 and M<>9)  
:Then  
:Goto K  
:Else  
:Goto 0  
:End  
:Lbl 0  
:If M=9  
:Then  
:Goto F  
:Else  
:Goto G  
:End
```

The program reminds you of your choice and sets the win and loss counters to 0. Then it prompts the user for the number of trials.

```
:Lbl F  
:0->L  
:0->W  
:Lbl A  
:ClrHome  
:Disp "STAY WITH FIRST"  
:Disp ""  
:Input "TRIALS: ", N
```

A loop is created to repeat the “remain with original” scenario N times. The two remaining doors are generated and the program assigns your original choice to the variable G and the winning choice to the variable W . The third door can be ignored since it has been revealed as a loss. Finally the program decides you win if your original choice matches the winning door and displays a result.

```
:For (I, 1, N)  
:int 3rand+1->G  
:int 3rand+1->B  
:If G=B  
:Then  
:W+1->W  
:Disp "WIN"
```

```

:Else
:L+1->L
:Disp "LOSE"
:End
:End
:ClrHome
:Disp "WINS=", W
:Disp "LOSS=", L

```

The user is given a choice to run another simulation or quit the program.

```

:Output (6, 1, "[ENTER] CONTINUE")
:Output (7, 3, "[ON] BREAK")
:Pause
:Goto K

```

A loop is created to repeat the “change mind” scenario N times. The two remaining doors are generated and the program assigns your original choice to the variable G and the winning choice to the variable W . The third door can be ignored since it has been revealed as a loss. Finally the program decides you win if your original choice matches the winning door and displays a result.

```

:Lbl G
:0->L
:0->W
:ClrHome
:Disp "CHANG MIND"
:Disp " "
:Lbl A
:Input "TRIALS", N
:For (I, 1, N)
:int 3rand+1->G
:int 3 rand+1->B
:If G=B
:Then
:L+1->L
:Disp "LOSE"
:Else
:W+1->W
:Disp "WIN"
:End
:End
:ClrHome

```


:Disp "WINS=", W
:Disp "LOSS=", L

The user is given a choice to run another simulation or quit the program.

:Output (6, 1, [ENTER] CONTINUE")
:Output (7, 3, "[ON] BREAK")
:Pause
:Goto K

Related Bookmarks

For a great description of the Monte Hall problem, visit:

Monte Hall (Let's Make a Deal) Problem

To see John P. Powers original Monte Program in its TI-85 format, visit:
[InterNIC's Graphing Calculator Programs](#)

To get a MacIntosh simulation, visit:
[Mike Naylor's Software Page](#)

[Return to Boyd's Homepage](#)

URL:<http://math.rice.edu/hemphill/Professional/Presentations/MonteHall/Monte.html>

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Testing a Mathematician's Mettle

A puzzle with connections to a realm of mathematics known as coding theory has garnered considerable attention in certain academic circles.

The Puzzle

Three players enter a room and a red or blue hat is placed on each person's head. The color of each hat is determined by a coin toss, with the outcome of one coin toss having no effect on the others. Each person can see the other players' hats but not his own.

No communication of any sort is allowed, except for an initial strategy session before the game begins. Once they have had a chance to look at the other hats, the players must simultaneously guess the color of their own hats or pass. *The group shares a hypothetical \$3 million prize if at least one player guesses correctly and no players guess incorrectly.*

Warning: Those interested in solving the puzzle themselves should consider skipping the partial solution.

A Solution

It is possible for the group to win three-fourths of the time because three-fourths of the time, two of the players will have hats of the same color and the third player's hat will be the opposite color. The group can win every time this happens by using the following strategy: Once the game starts, each player looks at the other two players' hats. If the two hats are different colors, he passes. If they are the same color, the player guesses his own hat is the opposite color. This way, every time that hat colors are distributed two and one, one player will guess correctly and the others will pass, and the group will win the game.

Using this strategy, the group will win when the hats are distributed two and one, but they will lose when all the hats are the same color.

(NEW YORK TIMES SCIENCE TUESDAY APRIL 10, 2001)

FIGURE THIS

Your team is down by one point. Your teammate, who makes free throws about three-fourths of the time, is at the free throw line. She gets a second shot if she makes the first one. Each free throw she makes is worth 1 point. If there is no time left, what are the chances you win the game?

Probability is a measure of chance. Applications of probability are found in genetics, the insurance industry, lotteries and medical testing.

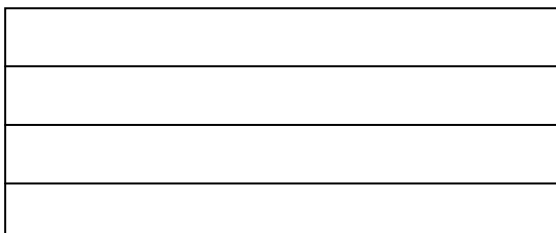
Answer:

The probability that your team wins is $9/16$, or about 56% of the time.

Complete Solution:

There are many ways to answer the question:

Make a rectangular diagram with four rows of the same size. Shade three of the four rows to represent making the first shot.



Make four columns of the same size and shade three of them to represent making the second shot.

There should now be 16 cells in the grid. The 9 cells that are shaded twice represent success on both shots which means your team wins without any overtime play. Since 9 of the 16 equally likely outcomes represents wins, the probability of winning is $9/16$.

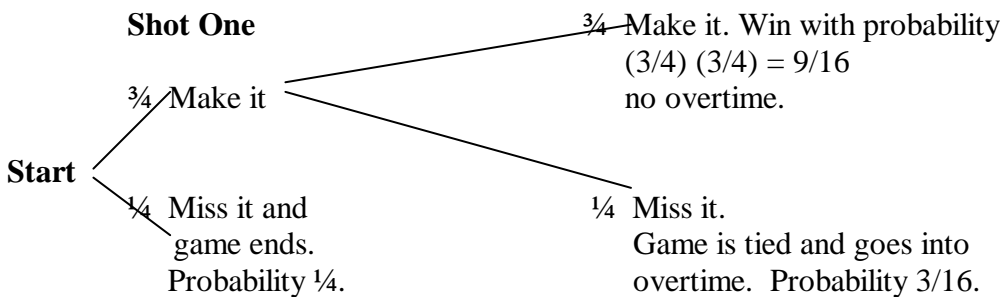
A different strategy is to draw a tree diagram labeled with all outcomes and their probabilities for each shot. The probability of winning is found by multiplying the probabilities on the appropriate branches of the tree.

If your teammate makes the first shot $3/4$ of the time, then $3/4$ of those times that she makes the first shot, she will make a second shot; that is:

$$(3/4) (3/4) = 9/16$$

In this case, the probability of winning without overtime is $9/16$.

Shot Two



[Multiplying probabilities here is correct only if the two shots are independent events. Assume that they are.]

(Figure This website: www.figurethis.org)

A DAY AT THE RACES

Let's go to Sam Houston Race Park for a day at the races. However, unlike the real program, we will assume every horse has an equal chance of winning. It is now time for Race 1.

First half of Daily Double, Win/Place/Show Exacta Trifecta, Superfecta Wagering

1 Texas/Maiden
Purse \$6,500

Number	Horse	Probable Odds
1	Fundamental Counting Principle	
2	Independent Event	
3	Tree Diagram	
4	Permutations	
5	Ven Diagram	
6	Dependent Event	
7	Combinations	

What is the probability of Horse 3 winning ? _____

You will not see a horses' probability listed on the program. What you will see are the odds against the horse winning. What are the odds on Horse 3? _____

In this race we will only concern ourselves with the Daily Double. The Daily Double is a bet that you can pick the winners of two consecutive races. If you pick two horses from the 1st race and three horses from the 2nd race, how many tickets will you have? _____

Second half of Daily Double

Pick 3 (Races 2, 3, & 4) W/P/S, EX, QU, TRI, SPR

2 Claiming
 Purse \$6,500

Number	Horse	Probable Odds
1	Circular Permutations	
2	Experimental Probability	
3	Odds	
4	Permutations with Repetitions	
5	Mutually Exclusive Events	
6	Inclusive Events	
7	Conditional Probability	
8	Event	
9	Outcome	

What is the probability of Horse 8 winning? _____

What are the odds on Horse 8?

Race Two has Exacta Wagering. The Exacta is a wager that your top two selections will finish first and second in exact order. If you like four horses in the race, how many tickets will you have to buy? _____

The Second Race also starts a RICK 3 for Races 2, 3, & 4. A Pick Three is a bet that you will pick the runners that will win three consecutive races. You like four horses in the 2nd Race, two horses in the 3rd Race and five horses in the 4th Race. How many tickets would you have to buy? _____

WIN/PLACE/SHOW, EX, QU, TRI, SPR WAGERING PICK 3 (RACES)

Number	Horse	Probable Odds
1	Circular Permutations	
2		
3		
4		
5		
6		
7		
8		
9		
10		

You like two horses in Race 3. If you decide to bet both horses to win, what would be your probability of winning? _____

Race Three also has Quinella Wagering. The Quinella is a wager that the two horses you select will finish first and second in any order. If you pick three more horses to go with the two you like giving you a total of five horse, how many tickets will you have to buy?

WIN/PLACE/SHOW, EX, QU, TRI, SPR WAGERING PICK 3 (RACES 4, 5, &6)

4 Claiming
 Purse \$5,400

Number	Horse	Probable Odds
1		
2		
3		
4		
5		
6		
7		
8		

Race 4 has Trifecta Wagering. The Trifecta is a wager that you can select the first three horses to cross the finish line in exact order. If you like four horses, how many tickets will you have to buy? _____

Probability Definitions

Conditional Probability

The probability of an event occurring given that another event has already occurred.

Event

One or more outcomes of a probability experiment.

Complement of an Event

All the events in the sample space except the given events.

Dependent Events

Two events are dependent if the first event affects the outcome or occurrence of the second event in a way the probability is changed.

Equally Likely Events

Events which have the same probability of occurring.

Independent Events

Two events are independent if the occurrence of one does not affect the probability of the other occurring.

Mutually Exclusive Events or Disjoint Events

Two events which cannot happen at the same time.

Experimental Probability

Uses a frequency distribution to determine the numerical probability. Experimental probability is a relative frequency.

Frequency Distribution

This proportion of the number of favorable outcomes over the total number of outcomes.

Inductive Probability

Uses probability values based on an educated guess or estimate. It employs opinions and inexact information.

Outcome

The result of a single trial of a probability experiment.

Sample Space

Set of all possible outcomes of a probability experiment.

Theoretical Probability or Statistical Probability

The ratio of the number of outcomes that make the event happen to the total number of outcomes. Uses the sample space to determine the numerical probability that an event will happen.

ANNOTATED BIBLIOGRAPHY

Boyd, Cindy J. Glencoe Algebra 2 Integration Applications Connections. New York, New York: McGraw-Hill. 1998.

This is a textbook used in teaching Algebra II. It has a chapter on Basic Probability Formulas. It is used by Houston Independent School District among other school districts.

Community Services Brochure

Darrell Huff and Irving Geis. How to Take a Chance. New York London: W.W. Norton & Company, Inc. 1959

This is an excellent book on Probability Theory. This book provides the rational behind Game Theory along with general information on the History of Probability.

Griffin, Peter. Extra Stuff Gambling Ramblings. Las Vegas, Nevada: Huntington Press. 1991.

This book is mainly concerned with probability as applied to gambling. It is a collection of articles he wrote for casino and sports which has ceased publication. Along with some new, expanded and updated articles assembled in book form.

Robinson, Sara. The New York Times Science April 10, 2001. New York: New York Times. 2001

Singh, Jagjit. Great Ideas of Modern Mathematics: Their Nature and Use. New York: Dover Publications, Inc. 1959

This book provides the layman with an understanding of the main mathematical ideas used in the sciences of modern civilization. However, it requires knowledge of high school mathematics. Chance and Probability is just one chapter.

Texas Education Agency

Publishes the Texas Essential Knowledge and Skills (TEKS) for all courses.

Internet Resources

Figure This:

www.figurethis.org

This website provides various math challenges for the family.

Hector Parr's Pages

www.C-PARR.FREESERVE.CO.UK/HCP/pro.htm

This is a website on various essays on Math, Physics, Cosmology and Philosophy.

The Monte Hall Problem:

www.MATH.rice.edu/~hemphill/professional/presentations/MonteH/Monte.html

This is a website associated with Rice University.