Probability, Statistics, and Figuring the Odds

Santosh Aggarwal

INTRODUCTION

I am a math teacher at Robert E Lee High School, located in the heart of Southwest Houston. It opened for classes in 1963, when the neighborhoods around Hillcroft and Richmond were predominantly white and Houston was a mainly segregated city. Lee High School's student population was predominantly high middle class Anglos. Thirty eight years later, the City of Houston and Lee High School have changed radically. The student body is populated with teenagers who come from 60 different countries to make this educational institution a truly UNO. These young adults speak 33 different languages. By the updated count, 75% students of the school are Hispanics and Spanish is the most widely spoken language among students. One of Lee school's best features is its E. S. L. Department. It accommodates nearly 50% student population and all newly arrived immigrants. Recently immigrated students in school have come with very little or no knowledge of English and are classified as limited English proficient. They are learning English as a second language. Due to reasons beyond their control, many students have had no formal education or very little education in their home countries. Many of these young adults are highly motivated and are trying their best to avail the opportunities and are making strides by advancing rapidly, but there are a few who have not and will not avail the opportunities. They come to school to socialize and to pass time because they are being forced to come to school. These students are absent more days than they are present in the school and are at risk of dropping out of school. These students add to the numbers of high drop out rate, high absentee rate, and high failure rate. To alleviate these and many other problems, the school has been changing to accommodate the different and diverse learning levels and interests of the students. When school opens in August 2001, Lee High School will have approximately 2,200 students enrolled in ten small learning communities. The learning communities are thematic and it is expected that the students with different learning interests and abilities will study in a community of their interest; be with the same group of teachers and students for four years of high school and successfully complete the high school education; be prepared for the college education; and be prepared for the changing job markets and job opportunities.

Probability, Statistics, Odds, and Stock Market

Fortunes are made and lost every day in the financial markets. The daily trading of stocks, bonds, and commodities generates a great many statistics, which experts must continuously process and analyze in order to assist investors. The boom of the last decade and the bust of the 2001 stock market are generating a lot of excitement, uncertainty, and anxiety not witnessed in the past half century. As the technology and related companies' share prices went up, up and away, the Chairman of the Board of the Federal Reserve Alan Greenspan commented, “There is an irrational exuberance prevailing in the market.” Statistical data did not support the proliferation of many technology companies and their bloated share-prices. The odds were stacked up against the investors. The boom went bust.

I will include the business section of Houston Chronicle in math curriculum to discuss the financial news that makes the headlines. Students will be exposed to the concept of Stock market Exchange and the existence of three different exchanges in the United States and many exchanges in the advanced countries of the world. Financially strong and highly visible companies such as General Electric, Microsoft, I. B. M. and Nike etc., whose products, the students and their families use on daily basis, will be introduced and discussed with the students. The students will be introduced to the listing of these companies and thousands of other companies on the New York Stock Exchange, the ticker symbols, the high and the low, and the extreme fluctuation in the stocks’ prices and the stock market. The students will search for the financial data and news of the companies and exchange information with the class. If the spark is ignited to know and understand more about the companies and the financial markets, the students will be advised to watch the Nightly Business Report program on Public Broadcasting Service, Monday through Friday at 6:30 P M.

Objectives:

The students will be exposed to real life applications and many uses of the probability and statistics and many careers where the use of Statistical Probability and Odds is extensive. Students will recognize that they have used probability in many decision making processes.

In the morning as we prepare to start our day, getting ready for school or work, and the weather seems questionable, we tend to tune in to the radio or turn on our television to watch and listen to the weather forecast. A Forecast of a steamy hot day, a freezing cold day, or a rainy day with the 95% probability of widespread flooding affects the way we dress for school or work.

The United States of America has the largest insurance and financial industry in the world**.** Millions of people work in this industry to offer services to the people of the United States of America and Americans living abroad. For people working in this industry, a sound knowledge of statistical probability and odds is very important because the insurance industry very heavily makes use of the concepts of probability, statistics and odds in insuring people, property, and their lives. For consumers like you and me, who pay for the advice and services of the insurance companies, it is important to understand the type of insurance we want, the deductibles, different options of the plan, and our level of tolerance in case of loss. We need not pay for the services that we might never need and never use.

E. F. Moody's Insurance overview states that, " Insurance properly utilized, is nothing more than a pool of money to reimburse those that have a misfortune befall them. The others simply get the emotional protection for a loss that never occurs. In essence, therefore, you hope to never use it."

Probability is expectation founded upon partial knowledge. A perfect knowledge leaves neither room nor demand for a theory of probabilities (Clarkson).

The concept of probability theory has been widely used and applied in the insurance industry. The premiums involved in any type of insurance are calculated or estimated based on the probability of occurrence of the relevant event. The actuaries working for the insurance companies are constantly juggling with the numbers. Their everyday job is to calculate risks and premiums for the insurance companies. They do extensive behavioral study and analyze characteristics of the behavior to calculate the premiums. The premium values are also obtained by taking a random sample of the same age group from the population and the risk involved in that particular age group. The more the risk involved and the reckless behavior, the higher the premium. The students will be exposed to the risks associated with their age group and the higher auto insurance premium they have to pay to obtain the auto insurance. For life insurance, the premium the young people pay is minimal, because the risk of death and life threatening diseases in this age group is minimal.

All students will participate in discussing the information on car insurance, medical insurance, life insurance, and property insurance. It is expected that they will not have life or property insurance but they will need insurance and information sometime in the near future. They will be asked to bring information on one type of insurance per group. They will be advised to seek information from their parents, friends, and other sources and share it with the class. Many students drive to school and to their work and have liability insurance. They will be asked to share information on the liability insurance, the premium they pay, and what precautions they will take to avoid higher premiums or cancellation of the policy.

Probability, Statistics, Odds, and Careers

In the medical profession, the physicians diagnose diseases by symptoms, by taking the history and physical of the patient, by examination, by considering all possible causes of the disease, and by the process of elimination and inclusion of causes and a host of other factors. Using statistical data and available information they may diagnose the disease confidently, prescribe the treatment, yet be uncertain of the outcome. In the medical profession the statistical data is extensively used and applied to calculate the statistical probability in making a diagnosis.

The unfounded belief of some people claiming a human life expectancy of 120 or even 150 years is not supported by any statistical data. The researchers analyzed trends in human life expectancy and presented the data supported by many years of research. They anticipate that many people here today will live long enough to witness a life expectancy of 85 years, but everybody alive today will be long dead before a life expectancy of 100 years is achieved. There are no magic potions, hormones, antioxidants, or biomedical technology that exist, that would permit a life expectancy of 120 years. The research indicates that the developed nations of the world would enjoy the most improvement in life expectancy. This information based on statistical data will be particularly helpful to remind students that they live in one of the most developed nations of the world and they need to practice and live a healthy life style to achieve the life expectancy anticipated by the outstanding researchers of modern time.

Statistical data indicates very high rate of venereal diseases among teenagers. The purpose of this unit will be to educate the students that these diseases are preventable with a little precaution. Untimely, unwanted pregnancy in high school reduces the probability of graduating from high school dramatically, and the dream of walking on the stage with a high school diploma lost forever.

Anticipating a life expectancy of 85 will have the Social Security Administration to review the social security tax deductions and payments to social security recipients who will be living longer. Many people who would think of retiring early might have to postpone retirement for sometime to qualify for full benefits. Accordingly, the retirement age has to be adjusted for the healthy working people to contribute longer to claim the retirement benefits longer. The domino effect creates the need for continuous adjustments in the system.

Applications and uses of statistical probability in medicine are far reaching. The scientists at the Center for Disease Control are evaluating the probability of contracting the disease if someone is exposed to a certain virus. Many of these evaluations are based on probability and involve the use of extensive statistical data, analysis, and interpretation.

An account of a mother whose 5 years old son had been diagnosed with Muscular Dystrophy; a serious, hereditary, chronic, progressively deteriorating, and x-linked disease was terrified by the news and she wanted to know the statistical data and the probability of her other children being afflicted or being carrier of the disease; and the probability of conceiving another pregnancy with these traits. The physician provided the information based on and supported by the statistical data so that informed mother could make the right decision. Statistical data and analysis of this genetic disease indicates that each pregnancy of a carrier mother of muscular dystrophy traits carries a 25% risk of muscular dystrophy disease. There are four possibilities of any pregnancy; a normal male, a normal female, a male with muscular dystrophy disease, and a female carrier of muscular dystrophy traits. Genetics, a subject of medicine that extensively involves statistical analyses of the data pertaining to hereditary diseases is an area where the students can explore the potential growth.

In the airline industry, the ticket booking agent makes reservations and despite being sold out, he is unsure of whether passengers with a reservation will be a show or a no show. To fix this uncertainty to a reasonable extent and to show maximum profit for the company, the ticket agent using his knowledge of statistics, probability, and odds, books a few extra seats as needed, charging a premium price. What and how if all passengers with reservations show up? Suggestions will be solicited from the students. Current practices used by airlines to solve these problems will be discussed. Airlines offer incentives to willing participants, who choose to take a later flight to accommodate others. In the reverse situation, of no show passengers, the tickets are already oversold and it is expected that no show passenger's seats will be occupied by the premium price paid passengers.

Advertising is a big business that gets its ideas from statistical probability. Students will be reminded of the free Coca-Cola bottles offered to all students at Robert E Lee High School in the year 2000 and the company's highly successful technique of advertising coke in the school. The Coca-Cola company is doing a roaring business in the school and an added income for the school. The students will understand that the money helps to pay for many projects of the school such as scholarships and fees paid for students to earn credit by exam, SAT, and PSAT exam and to purchase computers for their classrooms. Last year the competition between Pizza Hut and its rival Dominos Pizza in the school to advertise their pizzas to capture the pizza market was a very successful campaign. Students will be asked to participate in the discussion of techniques used by both companies and why and how Dominos Pizza was successful in capturing the pizza business in the school.

Some students may consider possible career opportunities associated with understanding trends, and collecting and analyzing large quantities of data – such as advertising.

People in all cultures enjoy games that involve chance or luck. They had a good time tossing the coins, throwing the dice and predicting outcomes to win the games. Inventing and playing games for entertainment and career has become a big business. Chess, an ancient game believed to have its origin in India about 600 years ago, is a game that requires a high level of concentration and a well thought out strategy. Recognized by the intellectual communities in many countries of the world, the game has attracted chess players to participate in the world championship. Its players command great respect, money, popularity and a legitimate place in the sports and games arena. Dominos is another game played throughout the world by the young and old alike. The proliferation of casinos in many big cities is a big business. Fortunes are rarely made in these glittering places but addicted players in the casinos lose everything including their sense of balance. However this entertainment industry provides a great number of jobs and a good source of entertainment if the players are not driven too much by the greed and instead play the game with knowledge and restraint.

The above mentioned careers are just a few of the host of professions such as judging sports, competitions, weather reports, and grading where the knowledge of statistical probability and odds is extensively applied.

Collecting Data for a Project

How many Americans are born every year?
Students all over the United States calculate the numbers using all sources and available data. This question involves students to collect statistical data from different sources, manipulate it using operations, and come to an approximate number. Last year, my Sophomore Students wanted to do a project for extra credit. I thought of a project that will involve all students to work in a cooperative learning environment. The next unit to study in Algebra class was on Probability and Statistics. I thought a project on statistics would be quite appropriate at this time. I wrote these two topics on the chalkboard.

How many children are born in Houston every year?
How many people die in Houston every year?

Two large groups with equal numbers of students in each group were formed and the students have had a choice to join the group they preferred. Initially students started guessing and giving any number that came into their imagination. I accepted and wrote all responses on the chalkboard. When asked, how they chose that number a few students could not come up with any explanation and others came up with reasonable explanation. After some brain storming, a few students asked, "How many people live in Houston?" I thought that was a good start. Next step was to think of where to look up for population record of Houston. Suggested sources were to search on Internet, Encyclopedia, newspaper, and Almanac etc. Some students were searching on the Internet while others went to the library to explore the information in the encyclopedia. A few of them asked the librarian's help to search for information in the almanac.

According to 1999 information Please Almanac Houston, Texas, estimated that the population of Houston County was 1,845,967. But that was last year. Students tried but they could not find population for year 2000. I brought the idea of census taken every 10 years to count all Americans for election and other purposes.

I went to the Public Library and looked in the Statistical Abstract 12th edition 2000 of the United States. I did not find any record of census 2000. However, Statistical Abstract showed births by states and Texas had 342,283 births and 142 deaths per ten thousand and the population of Texas was 20,044,000 in the year 1999. I brought this information for the students from the Public Library.
Students could estimate population of Houston for the year 2000 by adding births, subtracting deaths and calculate population using ratio and proportion. Some students suggested if we round population of Houston for the year 2000 it approximates to

2,000,000. The ratio of Houston to Texas population is 2,000,000 / 20,000,000 that simplify to 1 to 10 or 1 / 10, and 10% of the total population of the State of Texas. A student suggested,we could use 1 / 10 to find total births in Houston.

$$342{,}283 / 10 = 34{,}228.3.$$

Students recorded, approximately 34,228 children were born in Houston in that year. Let us calculate deaths by ratio and proportion, a student suggested.

$$142 / 10{,}000 = x / 2{,}000{,}000$$

It came to be 28,400 deaths for the year – approximately 28,400 people in Houston died in that year. Students recorded on chalkboard. This was students' project and they had good time searching, calculating, and estimating the accumulated data. They learned to collect relevant data from different sources, manipulated data to estimate, practiced different operations and concepts to calculate the numbers and finally to estimate the reasonableness of the numbers.

There are many ways and sources to find the number of births and deaths in a city in one year, but I thought this project was comprehensive, involved all students, and prepared students for the unit.

Prerequisite knowledge

Students should have an elementary knowledge of functions and variables. They should be familiar with properties of rational numbers. All probabilities lie between 0 and 1 inclusive. The students will asked to recall 10 numbers between 0 and 1 and write these numbers in decimals, fractions, and percents. A short discussion will follow to help students to understand the different names for the same number or value and why it is important to know these numbers in three different forms. The state-mandated exit level test states that students should be able to determine relationships between and among fractions, decimals, and percents. Students will complete warm up exercise to review fractions, reduce fractions, decimals, percents, and rational numbers. A through I are variables with values 1 through 9. Each value corresponds to exactly one variable. The variables are arranged in the triangle below such that the sum of the values of variables of each side is the same.

A + I + H + G = G + F + E + D = A + B + C + D

			A			
		I		**B**		
	H				**C**	
G		**F**		**E**		**D**

Students will complete the exercise using clues given below.

1.	**18 / 90 = A / 5**	**A =**
2.	**2 out of 8 or 1 out of B**	**B =**
3.	**90 % = C / 10**	**C =**
4.	**0.75 = D / 4**	**D =**
5.	**5 chances in 7 or 5 / E**	**E =**
6.	**36 / 60 = 3 / F**	**F =**
7.	**When flipping a coin, the chance of flipping a head is 1 out of G.**	**G =**
8. .	**When rolling a die, the chance of rolling a 5 is 1 out of H ,**	**H =**
9.	**2 / 3 = I / 12**	**I =**

Students will extend this knowledge by naming this formation an equilateral and equiangular triangle.

TASS and End of the Algebra Course Exam Objective

An objective of the state mandated TAAS exit level test states that the students will demonstrate an understanding of probability and statistics using counting procedures. Students will find the probability of simple and compound events, analyze the data and determine the mean, the mode, and the median of the data. This unit is written for freshmen students and will be reviewed for sophomore students to prepare them for the exit level TAAS test. The unit will be completed in two weeks, for a block-scheduled class of 90 minutes that meets everyday.

For students who have limited English proficiency, are academically unsuccessful, and unmotivated, the thrust of the lesson will be to engage students in groups in hands on activities such as tossing the coin, throwing a die, claiming success or failure as it occurs, and recording it. Students will understand that heads and tails are equally likely outcomes, but in practice on limited trials, heads and tails may not come up an equal number of times. Just because the coin came up heads on the first toss does not mean the coin will come up tail on the second toss. They will toss the coin 25 times and record the outcomes. Each group will report the number of heads and the number of tails of 25 tosses of the coin. Each group's heads count and tails count will be recorded on the chalkboard and added to analyze and interpret the outcome. The students will understand that for a very large number of trials, the outcomes will tend towards equivalence.

The class will be split into six groups. Each group will roll a die fifty times, and will keep a count of the outcomes. While the students will be rolling the die, a table depicting the possible outcomes of a roll of the die (1 through 6) will be drawn. Each group's number of rolls of 1's, 2's, 3's, 4's, 5's, and 6's will be added to show that each number has come up approximately 50 times and 50 divided by total number of rolls (300) equals 1/6. It will be explained that theoretically each number is expected to come up an equal number of times, but in experiments each number might not come up an equal number of times. However, the observed probability will be equal to the number of successes divided by the number of trials. At the end of the activity they will be able to understand that the roll of a die has six possible outcomes and each outcome has an equal chance of taking place. The six outcomes are the sample space of a roll of the die. On a limited number of trials (rolls) each outcome might not occur an equal number of times but for a large number of rolls, it is anticipated that all outcomes will take place just about equal number of times. Students will learn the terms associated with this topic in context and use them in describing the events and answering the questions. Many activities using coins, die, and spinners will be introduced and it is expected that students will be able to make connection of the ideas of the activities and skills with their real life experiences.

To understand **equally likely outcomes**, the students will play the Random Walk Game in groups of four students. Each group will be given a spinner divided into four equal sections labeled North, South, East, and West and a graph paper with a coordinate plane. The groups are to spin their spinners and then move one unit on the coordinate plane starting at the origin in the direction indicated by the spinners. They will spin 40 times. Before the groups will start, the students will be asked to predict where they think they will be on the graph paper at the end of the exercise. After 40 spins and moves each group will share their ending point on the graph paper with the class and analyze and interpret the result. It will be explained that because each outcome is equally likely, they should not have gone very far. The directions North South, East, and West cancel each other out. This is an example of how equally likely outcomes mean no outcome is favored, and each outcome has equal chance of taking place. Though our intuition may tell us that we should have moved away from the center or origin, the laws of probability say that we should not have.

Exploring Probability with two dice

The students will roll two dice several times to see all possible outcomes. To understand the counting principle, the sample space, and the sums of all throws of two dice, they will list all the outcomes in a systematic way. The chart below will help them understand the 36 outcomes and 11 sums and the frequency of each sum.

	Red Die	**and**	Black Die		
(**1** , 1)	(**1** , 2)	(**1** , 3)	(**1**, 4)	(**1**, 5)	(**1**, 6)
(**2**, 1)	(**2** , 2)	(**2** , 3)	(**2** , 4)	(**2** , 5)	(**2** , 6)
(**3** , 1)	(**3** , 2)	(**3** , 3)	(**3** , 4)	(**3** , 5)	(**3** , 6)
(**4** , 1)	(**4** , 2)	(**4** , 3)	(**4** , 4)	(**4** , 5)	(**4** , 6)
(**5** , 1)	(**5** , 2)	(**5** , 3)	(**5** , 4)	(**5** , 5)	(**5** , 6)
(**6** , 1)	(**6** , 2)	(**6** , 3)	(**6** , 4)	(**6** , 5)	(**6** , 6)

The set of all possible outcomes is called the **sample space**. 36 outcomes listed above can be calculated by using the **Fundamental Counting Principle.**

Fundamental Counting Principle If event A can occur in x ways and is followed by event B that can occur in y ways, then the event A followed by event B can occur in x y ways.

number of outcomes for first die		number of outcomes for second die		number of possible outcomes
6	x	6	=	36

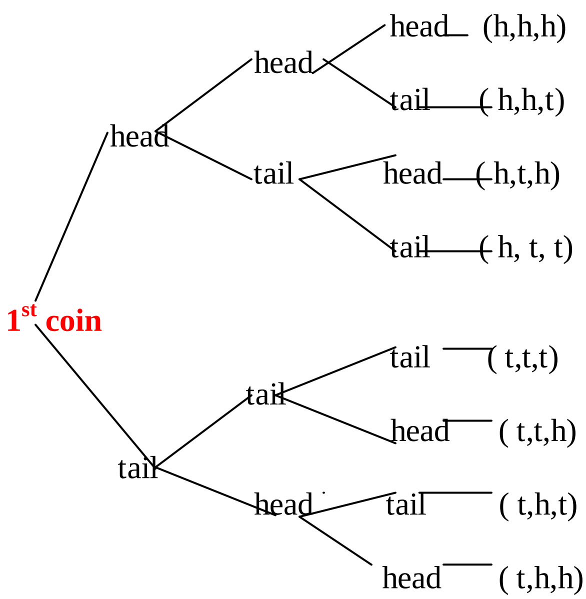
It is anticipated that the students will get frustrated counting and listing the outcomes of three dice.
They will be suggested to apply the Fundamental Counting Principle.

outcomes of first die		outcomes of second die		outcomes of third die		number of total outcomes
6	x	6	x	6	=	216

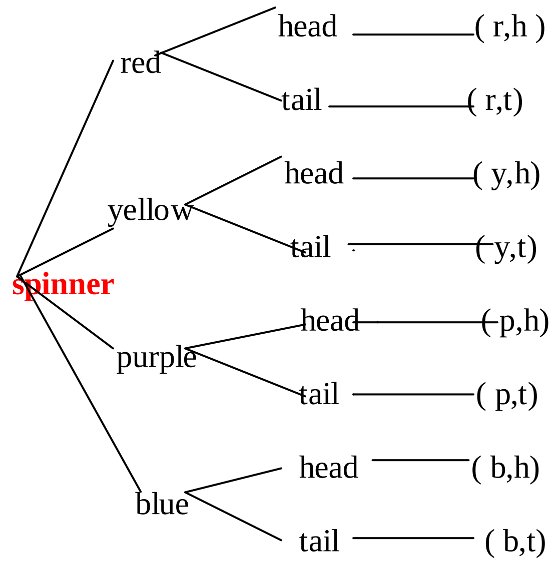
Tree diagram

Another way to understand and visualize all the outcomes of two or three or any number of events is by a tree diagram.

toss three coins all at once
outcomes of 3 coins



spin a spinner and toss a coin
outcomes of a spinner and a coin



Students will calculate the outcomes of these two events by applying the Fundamental Counting Principle.

Odds

The students will be acquainted with the rudimentary knowledge of odds. They will discuss the phrases, odds of winning a lottery, odds of passing a test, and odds of surviving a crash or a deadly accident. Students will be asked to look up several meanings of odds in the dictionary and suggest the meaning used in reference to the statistical probability. The following sentences will be written on the chalkboard and discussion and explanation will follow to help students understand the concept of the odds and the probability.

On a toss of a coin, the odds in favor of coin showing up head is 1 to 1, and the probability of showing up a head is 1 to 2. On a throw of a die, the odds in favor of getting a 6, is 1 to 5, and the probability of getting a 6 is 1 to 6. The odds in favor of getting a number less than 3 on a throw of a die is 2 to 4 or 1 to 2, and the probability of getting a number less than 3 is 2 to 6 or 1 to 3.

25 students in a classroom consisting of 10 girls and 15 boys play a lottery game and each student buys one ticket of lottery game. The odds in favor of girls winning lottery will be 10 to 15 or 2 to 3, and the probability of girls winning the lottery will be 10 to 25 or 2 to 5.

Students will understand that another way to describe the chance of an event taking place is with **odds**. Probability can be calculated or measured in a different way called **odds.** Odds of a successful outcome are expressed as the ratio of

Successes / failures or s / f

Successes are the number of ways the event can occur and failures are the number of ways the event cannot occur.

Odds in favor of an event = s / f

The probability of an event is successes / (successes + failures) or sample space.

Probability of an event = s / (s + f)

Students will conclude that another way to describe the chance of a favorable outcome is with odds and odds in favor of an event have a better numeric value than the probability of the same event.

Simulation:

A simulation is an imitation of a given problem. Astronauts experienced gravitational differences on the moon. Lack of gravity on moon allows things to float in space. One-sixth gravitational effect on things and people has been simulated at NASA to train the present and would be future Astronauts. Simulation is acting out the event by modeling a situation. Airplane pilot students receive initial flying training in the simulated environment.

Students will understand the concept of how the real world events can be simulated by using dice, coins, and spinners and the simulation of events have the same number of outcomes as the number of outcomes when you actually do the experiment. This concept will be explained to students by writing a quiz of 10 true false questions. Since there are only two choices to answer the question, a coin will serve a good simulating device. Toss of a head will be recorded as f meaning the information given in problem is false and the toss of a tail will be recorded, as t meaning the information given in the problem is true. At the completion of the quiz, the recorded answers will be checked with the key answers. In quizzes of true or false choice, doing a simulation did not produce the passing grade. It is expected that the quiz just completed will not produce the passing grade of 7 correct answers. The students will be convinced that guessing the answers is probably not a good way to take a quiz or a test. However simulations are an integral part of many explorations where the time, money, manpower, and resources are at premium. The students will understand that the situations that require a large number of trials or experiments that might require weeks to obtain the results, to do simulation on a computer is an excellent way to generate the needed information instantly.

Students will roll a die 100 times and keep a record of the outcomes and the number of minutes it took to roll the die 100 times. They will run a computer program that will simulate rolling a die 100 times. Students will find that in the fast paced world and information technology age, to find probable solutions by simulating the results using computer is absolutely necessary. Simple computer programs of simulations will be shown to students to use and learn to do simulations.

The experts in casinos do many simulations to determine the probability of generating money for the casinos. Gambling is another pursuit where the simulation helps the experts to calculate the probability of profitability to the gambling industry. Students will understand that gambling is designed for the gamblers to lose, and that the odds against winning are stacked up.

Random Numbers in simplified terms are unbiased numbers selected from the entire data called the population. The Random numbers represent the whole population and are used to analyze and describe the data. Students will learn different methods of selecting random numbers and will learn to evaluate the reasonableness of the method used in producing a random sample. They will determine if the collected sample is representative of the entire population and is completely unbiased.

Five lucky winning numbers of Texas Lottery Games are random numbers. They are generated randomly from the whole population.

Measures of central tendency

Measures of central tendency are the numbers that represent a set of data and the three averages termed the mean, the mode, and the median interpret the data set.
Students use statistics everyday when they are talking of their grade average. Average grade, average weight and height, and average age of 9^{th} grade students are everyday examples of the term mean. A set of data may contain hundreds of items. Students are expected to understand that in order to understand the data, it is important to have a single number that represents the whole data items. This single number can be the mean, the median, or the mode of the data set.

Which “average” will best represent the data?
Each average has a meaningful place in different sets of data.
Grades on 1st test: 78, 74, 69, 72, 82, 68, 78, 80, 66, and 70
Grades on 2nd test: 98, 34, 78, 80, 78, 76, 78, 76, 78, and 78
Grades on 3^{rd} test: 40, 74, 72, 92, 72, 76, 24, 88, 28, and 90

Students will check the data and interpret it by the average most representative of the data set.

Each set of data has some interesting facts to analyze, and interpret. Having examined each set of data, the students will be able to interpret that the first data set will be best explained by the term **Mean**. All numbers cluster around some number. The number should be calculated to represent the data. The range of the data set is not very large.

The second set of data can be best explained by the average called **Mode**. One number in the data set is repeating frequently. This number represents the data set.

The third set of data has extremes, on the upper end and on the lower end. The range of the data is very large. Average of this data can be best described by the term **Median.**

A small company presents its salary data of all employees.

Employees	**Frequency**	**Salary**
Director	**1**	**$200,000**
Chairman	**1**	**$175,000**
Employees	**15**	**$35,000**
Maintenance	**1**	**$12,000**

Job advertisement: "Will you like to work for our company? Excellent pay, come join our Company."

The chairman calculated that the average salary of the employees is **$52,941.20.**
The employees calculated that the average salary of the employees is **$35,000.00**.
Students will discuss the two averages calculated.

A short test will be given to check the understanding of the measures of central tendency.

1. A set of data has a mean of 7, a median of 6, and a mode of 8. Write a data set for the averages.
2. Which average must be in the data set and why?
3. Modify your data so that the mean and the median of the modified data are 7 and the mode is 6. Students will tell whether the mean, the median, or the mode is being used. Explain.
4. The most popular shoe size is 7 ½.
5. Half of the homes were priced at $150,000 or less.
6. The average bill for a customer at Taco Bell is $3.69.p
7. The best-selling T-shirt colors were white and navy.

8. On the final exam, 75% students made a grade of 80 or better.
9. Including tests, quizzes, class work, and homework, my final grade for the first semester is 85.

Quality Control Engineering Profession will be discussed with the students with special emphasis on the knowledge of statistical probability and odds required for the job specification.

A quality control engineer at ABC Ever-glow Bulbs factory tested 400 bulbs and found 6 bulbs defective. The factory produces 500,000 bulbs a day. How many bulbs are likely to be defective?
What is the probability that a bulb selected at random will be defective?
What are the odds of selecting a bulb that is not defective?

Every thing in life is not fair. So are some of the games we play. Students will be expected to check the fairness of the game before they agree to play. Class will be divided in two groups. Two dice are needed to play this game. Each group will take turn to toss two dice, and will find the product of numbers of the dice. If the product of the numbers will be even, group A will scores a point. And if the product of the numbers tossed will be odd; group B score will score a point. The group with the most points at the end of 18 rounds will be the winner. A round consists of each group tossing the dice once. Students will make a list of all possible outcomes and answer the questions: Which group do you think will win? And why? Do you think each group will have an equal chance of winning? Explain. How would you change the game to make it fair?
This will offer an opportunity to explore and think of strategies to find solution that is fair for all players.

Students will play a game with a die and a spinner. Two groups will be formed with a toss of a coin; a Spinner Group and a Die Group. One group will be in control of die and the other group will be in control of a spinner. Explain to players that a game consists of one roll of a die and one spin of the spinner. Each group will take a turn. The die group will roll the die and the spinner group will spin the spinner. The spinner has three equal dividers with numbers 0, 5, 6 on the dividers. The die is six sided and has six numbers. On a roll of a die, the number it shows up will be number of points and the die group will add up the points of each roll. On each spin the number the spinner group spins will be their points and they will add up the points of each play. At the end of 18 games, the group that will accumulate the greater number of points will be the winner. Students will be given a chance to explore the activity and then they will answer the questions
If you will have a choice of a die or the spinner, what will be your choice, and why? Which group will have the greater chance of winning? What would you need to know in order to have enough information to win? How many games would you play before deciding which if either group has an advantage? How many possible outcomes are of this game? What is the sum of all throws of the die? What is the sum of all spins of the spinner? Are you convinced that one is better than the other is? Explain your finding

A casual look tells that the die has a better chance of winning, but the fundamental counting principle and the sums of all outcomes show that the spinner has a better chance of winning. There are 6 outcomes of the die and 3 outcomes of the spinner.
Using the Fundamental Counting Principle, the possible number of outcomes is 6 x 3 = 18. The Sample Space of this game will be 18. Each outcome is equally likely and no outcome is favored, there are 18 outcomes of the die and 18 outcomes of the spinner. The sum of the 18 outcomes of spinner is 66 and the sum of the 18 outcomes of die is 63. The chart below explains it.

Spinner and a Die

(0 , 1)	**(0 , 2)**	**(0 , 3)**	**(0 , 4)**	**(0 , 5)**	**(0 , 6)**
(5 , 1)	**(5 , 2)**	**(5 , 3)**	**(5 , 4)**	**(5 , 5)**	**(5 , 6)**
(6 , 1)	**(6 , 2)**	**(6 , 3)**	**(6 , 4)**	**(6 , 5)**	**(6 , 6)**

The discussion will follow and students will understand that all solutions are estimates and are calculated according to the probability theorem and the outcomes are counted using statistical probability.

The probability is the numerical measure that indicates the frequency or likelihood for a particular outcome to occur. Probability does not tell what number will occur on a roll of a die and on a spin of a spinner but indicates what is likely to occur and with what frequency will spinner group always be the winner? No, but the probability is that the spinner group will win. Can a die group ever win? Yes, with less statistical probability. Students will repeat this game to check the sum, and draw their own conclusions.

Venn diagram:

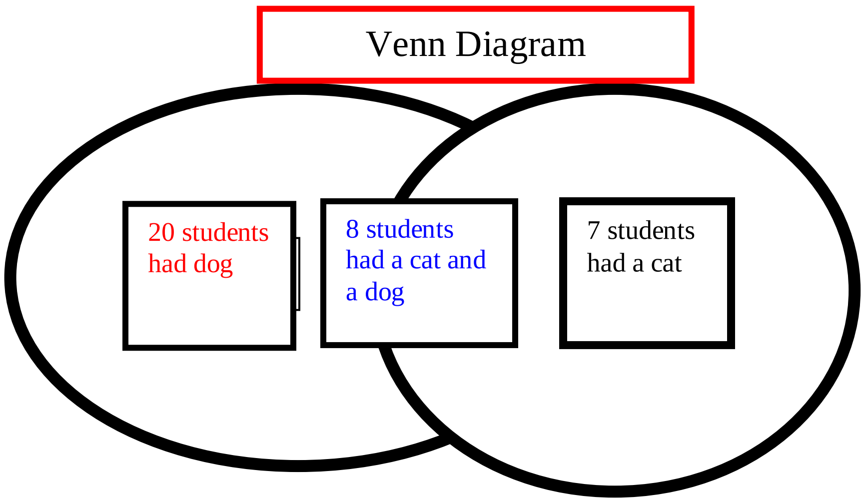
Finding probability using Venn's diagram is a part of the TAAS objectives.
I hope to cover this section of the unit with the students. Students on a survey conducted will tell if they have a dog or a cat or both (a dog and a cat). The response on last year's survey was, 28 students had a dog, 15 had a cat, and 8 had both, a dog and a cat. What is the probability that a randomly selected student will have only a cat?

Venn diagram is very helpful in visualizing the collected data placed in circles.
It helps students see the number of students who have only a dog and the number of students who have only a cat, and the number of students who have both.

Students who have a dog = 28
Students who have a dog only = 28 – 8 = 20

Students who have a cat = 15
Students who have a cat only = 15 – 8 = 7
Students who have both pets = 8

Total Number of students with pets = 28 + 15 – 8 = 35 or 20 + 7 + 8 = 35
Probability of randomly selecting a student who has a cat is only 7 / 35 = 1 / 5



A technique called " Tag and Recapture " will be introduced to estimate the total number of people, animals, birds, and objects etc.

A bag full of clear marbles containing more than a thousand marbles will be used in this experiment. Students will predict the number of marbles in the bag. The prediction will be recoded. They will take out 50 marbles from the bag and mark these 50 marbles with a red marker. They will put red color marked marbles back in the bag, and will shake the bag thoroughly. They will take out 50 marbles at random from the bag again. Fifty marbles taken out will be representative of the entire number of marbles in the bag. If in this sample of 50 marbles, 2 are found to be red color marked, then what would be the total number of marbles in the bag?

T = total number of marbles in the bag　　　　t = marbles drawn (recaptured)
N = total marbles marked red (tagged)　　　　n = colored marbles (recaptured)

$$\frac{T}{N} = \frac{t}{n} \qquad \frac{T}{50} = \frac{50}{2}$$

$$2\,T = 2{,}500 \qquad T = 1{,}250$$

Students will be explained that an experiment is an estimate and needs to be performed several times to get an accurate estimation. Students will be asked to perform the above experiment and collect data of each experiment to use it to find the mean, the mode, and the median and use these averages to explain their finding with statistical accuracy.

Students will find the answer to the question below and will connect the statistical probability associated with the solution of this problem.

What is the likelihood that two presidents of the United States share the same birthday?

Students will get involved in searching and making a list of 41 presidents and recording their birthdays. They will find out that there were two presidents born on November 2 and another two were born on December 29. It is important to note that in a group of 24 people, the probability that two people will share the birthday is 50%. Statisticians with calculations and experiments had determined this magic number 24. Students will collect the information of shared birthday from each of the four classes they attend and share it with all students to prove or disprove the validity of the number 24.

Professor Michael Field played a statistical probability game in the seminar that I found to be tricky and amusing. A dollar bill and five envelopes were used to play the game. The dollar bill was placed in one of the five envelopes. We were to choose one envelope of the five. According to the probability theory, each envelope had 1/5 chance of containing a dollar bill. We picked one envelope at random hoping it will have a dollar bill. Prof. knew which envelope has the dollar bill. He carefully removed three envelopes of the four remaining, assuring the removed three envelopes do not have dollar bill. Prof. gave us a choice to exchange the envelopes. We refused. We rationalized that the probability of a dollar bill in each envelope is fifty-fifty. Prof. opened his envelope and showed the dollar bill in his envelope. He won this game. He repeated this activity several times and we invariably did not exchange the envelope. Approximately, 4/5 times, Prof. won the game. At the end, it was explained that the probability after removing three envelopes had changed. Probability of the envelope we picked, remained the same, 1/5, but the probability of the envelope in his possession had increased from 1/5 to 4/5. I will to play this game with my students to convey the concept of changed probability associated with this game.

By involving students in games, activities, discussions, and projects, I hope the students will be able to understand the many uses of probability, odds, and statistics in their life. They will be able to make decisions of uncertain situations in a more effective way by utilizing the concepts and skills learnt in this unit. As consumers, they will reap the benefits of being informed citizens and may consider many job opportunities associated with this subject.

BIBLIOGRAPHY

Glance, *Building Bridges to Mathematical Excellence, A Transition to Algebra*, 1999.

McDougal Littell, *Mathematical Connections , A Bridge to Algebra and Geometry*, 1999.

South-Western Mathematics, *An Integrated Approach* , 1998.

Prentice Hall, *Algebra 2 with Trigonometry*, 1990.

Prentice Hall Mathematics, *The Fine Art of Teaching*, 1999.

National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics*, 1987.