Creativity and Mathematics

David R. Grai

PROBLEM SOLVING AND CREATIVITY IN MATHEMATICS

Typical Classroom Mathematics

In most elementary and secondary school mathematics classes math is taught to students as if it is a complete and unchangeable body of knowledge, with all rules and procedures well established. It is presented to students as if it were a "dead" subject, and it is no wonder that students deaden their minds and simply endure most of the mathematics they are required to take in school. According to King:

All of us have endured a certain amount of classroom mathematics. We lasted, not because we believed mathematics worthwhile, nor because, like some collection of prevailing Darwinian creatures, we found the environment favorable. We endured because there was no other choice. Long ago someone had decided for us that mathematics was important . . . so we were compelled into school classroom fronted with grey chalkboards and spread with hard seats. . . The room in which we sat was a dark and oppressive chamber and we thought of it then and now as Herman Melville thought of the Encandatas: only in a fallen world could such a place exist. (King, 15)

The focus of the mathematics taught is often on its application in real world situations, although most of the "real world" problems found in the textbooks are highly contrived and have little to do with reality. Mathematics is actually a changing, growing body of knowledge, created (or some would say discovered) by real people. Students need to see how mathematics was developed, and realize that throughout history creative individuals shaped the body of knowledge that we call mathematics. Again, according to King : ". . . research in pure mathematics means to produce new mathematics." Pure mathematics is the theoretical mathematics which can be derived, using the rules of deductive reasoning, from the few basic axioms that are agreed upon. It is math for math's sake does not include any "real-world" applications of mathematics. King then expands on the nature of pure mathematics:

The word "produce" as used here seems slightly awkward and it would be more natural to replace it with the word "create" or "discover.' But I have used the word produce . . . to describe the work of mathematicians because it is a continuing controversy in mathematical circles as to whether new mathematics is created or discovered. . . The idea that mathematics, as a physical world seems to exists, independent of human thought and activity is a notion at least as old as the philosophy of Plato . . . a second view claims that mathematical structures are created and that they have no existence independent of the person that created them. This notion meshes well with the nature of modern pure mathematics. (King, 41)

Pure mathematics is valuable in and of itself, and there is satisfaction in solving a difficult problem with a solution that is often unexpected and usually elegant in its simplicity. If it were possible for students to experience the beauty of mathematics, and the joy of mathematical creation at their own level, it is probable that many students would continue into the higher levels of mathematics, and do so with confidence. Most people who do not understand mathematics do not know that there is any pleasure in working with mathematics. For many years I taught in middle schools where interdisciplinary teaching units were used frequently. A topic wold be chosen, and teachers from the five subject area (English, reading, science, history, and mathematics) would plan a unit of study, which would often be culminated with a field trip. Lots of creative ideas would come up for every area except math. The other teachers suggested math activities such as counting and keeping track of the money collected for the field trip, or measuring and cutting the posterboard to make some display. The other subject teachers assumed that all of us could see the interesting and creative opportunities that were available in their subject areas, could only see math as being useful for its applications. Unfortunately, I didn’t feel I was going to change their minds, so I usually endured the interdisciplinary units and tried to make math enjoyable and challenging in my own teaching (when we were in between interdisciplinary units). What those who do not know mathematics often do not understand is that there is joy in the study of mathematics for its own sake just as sure as there is value top art, music, poetry, and the study of history. The joy of mathematical creation is not unlike the joy of writer's or artist’s joy in creation.

> Robert Frost once lived in Vermont. He lived among farmers. While they worked the land he worked at poetry. The farmers broke ground with ploughs drawn by horses. Frost drew verse straight as furrows across blank pages. . . Frost was touched in a way the farmers were not. He saw something the farmers could not see and could not appreciate . . . an abstraction and metaphor where the farmers saw only reality . . . and ordinary pasture becomes an irresistible invitation:
>
> I’m going out to clean the pasture spring
> I’ll only stop to rake the leaves away
> (and wait to watch the water clear, I may)
> I shan’t be gone long - you come too.
>
> Robert Frost saw in ordinary things values the farmers did not see . . . he understood, as do all true artists, that it is metaphor and symbol, and not plain reality, that is memorable and significant. Mathematicians, like poets, see value in metaphor and analogy. The lines they draw are made, not only of

> words, but of graceful symbols: summations and integrals, infinities turning on themselves like self-swallowing snakes, and fractals like snowflakes that, as you blink your eye, turn to lunar landscapes. Mathematicians write their poetry with mathematics. (King, 11)

Just metaphor and symbol can be found in what seem to be ordinary events, mathematics can be seen in almost everything around us, if we only take time to look. As John Allen Paulos states, "It's time to let the secret out: Mathematics is not primarily a matter of plugging numbers into formulas and performing rote computations. It is a way of thinking and questioning that may be unfamiliar to most of us, but is available to almost all of us." (Paulos, 3). In his book *A Mathematician Reads the Newspaper* Paulos highlights the mathematics embedded in different aspects of politics, economics, social issues, lifestyle and entertainment, medicine, environmental issues, food and diet, literature, and even obituaries. The book is not about mathematical applications, but rather about the analysis of situations using logical theoretical (pure) mathematics.

Students at all levels would benefit by being challenged by difficult problems (at their level) and being given time to work on the solutions. What is usually done is just the opposite - students are given the algorithm (no idea where it came from) and told to apply it to sets of problems. This makes mathematics seem trivial and removes all creativity from the process. This method of teaching is justified because students are told that they can apply the mathematics they are learning to real world situations. Those who do discover the beauty of mathematics often do so in a college course, perhaps their third year in college. I recall my own grade school and high school mathematics experiences. Never did I find anything pleasurable or stimulating. Some teachers made it less miserable than others, but for the most part I really had no idea what the purpose was, beyond simple applications which I could easily figure out for myself. On the other hand, I loved sports and constantly kept records of sports statistics. I kept my own batting average up to date when I played baseball. I also studies sports records for all of the major league sports, and also kept statistics of all my favorite players. I recall once in high school explaining to other players how to calculate the distance from home plate to second base using the Pythagorean Theorem. I certainly was capable of using mathematics, but in school I usually received grades of "C" or "D" in the subject. I also had an older cousin whose husband used to bring math riddles and puzzles to me. I always loved doing these, and was quite successful, but at the time I did not know there was anything mathematical about them. It was in my third year of college while attending the University of Michigan that I discovered that there could be pleasure in the study of mathematics. Most people stop pursuing mathematics long before this point, and those who make it this far often discover the value of pure mathematics quite by accident:

> Most of the mathematicians of this generation will admit that they came to their profession quite by accident . . . just by chance, we took a course outside our curriculum. We enrolled in a post-calculus course in analysis. And, unexpectedly, we encountered pure mathematics. We were struck by the subject, like Saul on the road to Damascus. Nothing in our backgrounds had prepared us for anesthetics of mathematics. . . it was a moment of great discovery. It was as though we had lived all our lives in the hold of some great ship and now were brought on deck into the fresh air. And we saw the unexpected sea. We felt, with W. B. Yates:
>
> All changed, changed utterly:
> A terrible beauty is born.
>
> The moment was worth the wait but we would be forever aware that we had come to it by chance. And along the way there had been many dropouts. (King, 19)

Improved Mathematics Teaching

There are three goals in this unit:

(1) To put mathematics into an historical context so students can have an understanding of where the math they learn comes from, and how it has been developed.

(2) To allow students to "derive" formulas and algorithms rather than simply giving the information to them to be applied.

(3) To give students the opportunity to attempt to solve mathematics problems which are interesting and challenging in and of themselves, even though they may have no real-world application.

Mathematics in an Historical Context

Mathematics is a growing and changing body of knowledge, and it is important for students to know where mathematics comes from in order to help them better develop their own understanding of the subject and their own role in creating that understanding. According to Schoenfeld:

> Mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within. Although the language of mathematics is based on rules that must be learned it is important for motivation that students move beyond the rules to be able to express things in the language of mathematics. This transformation suggests changes both in curricular content and instructional style. It involves renewed

effort to focus on:

- seeking solutions, not just memorizing procedures
- exploring patterns, not just memorizing formulae
- formulating conjectures, not just doing exercises

As teaching begins to reflect these emphases, students will have opportunities to study mathematics as an exploratory, dynamic, evolving discipline rather than a rigid, absolute, closed body of laws to be memorized. They will be encouraged to see mathematics as a science, not as a canon, and to recognize that mathematics is really about patterns and not merely about numbers. (Schoenfeld, 335)

By putting mathematics into an historical context students will have a better of understanding of where mathematics comes from. Polya relates ". . . a time when he was a student himself, a somewhat ambitious student, eager to understand a little mathematics . . ." Polya continues:

> He listened to lectures, read books, tried to take in solutions and facts presented, but thee was a question that disturbed him again and again: "Yes, the solution seems to work, it appears to be correct, but how is it possible to invent such a solution? . . . how can people discover such facts? And how could I invent or discover such things by myself?" (Polya, vi)

Students will learn that great thinkers have struggled with mathematics, and in overcoming their difficulties have created new and valuable aspects of mathematics. Some possible areas for student exploration are:

- Finding formula for the summation of integers - read about the childhood accomplishment of C. F. Guass who, as a ten year old child created a method for adding any set of numbers from one to any whole number. He developed this formula in response to a punishment he was given by his fifth grade teacher, who told the students to find the sum of all numbers from one to one hundred. Students could be given the same assignment, and allowed to struggle with it. Perhaps some of them will also discover a creative method of solution.

- The Seven Bridges of Konigsberg will give students an opportunity to explore a problem that has no solution, but the problem inspired Leonard Euler to develop a theorem for networks that allowed him to prove the problem unsolvable, a problem that mathematicians had been working on for more than three hundred years before it was finally solved.

The Creation of Mathematical Algorithms

Allowing students to derive formula and algorithms involves them in becoming creators and active producers of mathematics rather than acting as passive consumers, which is often the case in the mathematics classroom. Polya notes:

> A great discovery solves a great problem but there is a grain of discovery in the solution to any problem. Your problem may be modest, but if it challenges your curiosity and brings into play your inventive facilities, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime. This is closely related to the previous topic, for in developing formulas and algorithms the students will often mirror methods used when these formulas were originally developed. (Polya, v).

A teacher, according to Polya, should not fill the classroom with routine drill and practice, but rather challenge the students by giving them appropriately challenging problems for their level. Some possible activities for students would include:

- Being asked to develop formula for area of a triangle, trapezoid, or any other geometric shape. This can be done using an inductive approach, along with the use of a geoboard (a wooden board with rows and columns of small nails, often five by five, on which rubber bands are stretched to make geometric shapes. There are also plastic versions of geoboards).

- By solving non-routine problems involving topics such as topology, combinatorics, or discrete mathematics. An example, again using a geoboard, will be to discover Pick's Rule for area of a polygon on a square grid matrix.

Solving Non-Routine Problems

Challenging students with difficult problems which will capture their interest will help them to learn to appreciate the beauty of mathematics. The problems should be interesting, non-traditional, and have a variety of methods of solutions. The solution should not be easily predictable.

> Studying the methods of solving problems, we perceive another face of mathematics. Yes, mathematics has two faces; it is the rigorous science of Euclid, but it is also something else. Mathematics presented in the Euclidean way appears as a systematic deductive science, but mathematics in the making appears as an experimental inductive science. Both aspects are as old as the science of mathematics itself. But the second aspect is new in one respect:

> mathematics in the process of being invented has never before been presented in quite this manner to the students . . . (Polya, vii).

Mathematical problem solving has been a central and focusing issue in much of the literature of *The National Council of Teachers of Mathematics* (N.C.T.M.) during the past two decades. In the *Curriculum and Evaluation Standards for School Mathematics* the N.C.T.M. choose problem solving as the first of the thirteen topics (called strands) in which changes are recommended. By the time students reach high school, problem solving should not be a separate topic in the mathematics curriculum, but a central part of every topic studied. According to these *Standards*:

> Mathematical problem solving, in its broadest sense is nearly synonymous with doing mathematics. Thus, while it is useful to differentiate among conceptual, procedural, and problem-solving goals for students in the early stages of mathematical learning, these distinctions should begin to blur as students mature mathematically . . . problem solving is much more than applying specific techniques to the solution of classes of word problems. It is a process by which the fabric of mathematics is both constructed and reinforced. (NCTM, 129).

"Problem solving" is often confused with "application" in mathematics. One of the values of mathematics is that it can be applied to so many real world situations, even to the point where some believe that every physical situation can be described by a mathematical model. But problem solving does not have to remain relegated only to the area of real world application, although it begins this way for younger students. According to the N.C.T.M. *Standards*:

> One consequence of students' increasing mathematical sophistication is that problem situations, which for younger students necessarily arise from the real world, now often spring from within mathematics itself. Thus, mathematical problem solving serves not only to answer questions raised in everyday life, in the physical and social sciences, and in such professions as business and engineering, but also to further extend and connect mathematical theory itself. Problems and applications should be used to introduce new mathematical content, to help students develop both understanding of concepts and facility with procedures, and to apply and review processes they have already learned. (NCTM, 131)

"Problems have occupied a central place in the school mathematics curriculum since antiquity," Schoenfeld states, "but problem-solving has not. Only recently have mathematics educators accepted the idea that problem solving deserves special attention." (Schoenfeld, 337). The role of problem-solving has become more prominent in the school curriculum, especially in the elementary schools, where this trend started in the early 1980's. Creative problem solving is going on in many more middle school classrooms now that twenty years ago, but judging from the responses of the high schools I presently teach, the majority of the students are still getting a "drill and kill" type of instruction in middle school. Based on my own personal experience, students in sixth grade were much more enthusiastic and showed a higher degree of creativity when attempting to solve non-routine problems, than were the eighth graders. It gets worse in high school, where the mind set of the student set on applying algorithms and following rules. Fore the most part my students think of math class as a place where the teacher demonstrates some steps to get an answer, and then the students imitate the steps and try to get right answers. While I rarely had problems getting sixth grade students to take a broader view of mathematics and to put more of themselves into it, I have had little success in changing the views of my high school students. The activities in this curriculum unit can be used with middle school students as well as high school students. For high school teachers, these activities should not be taught as a unit, but worked into the curriculum at strategic points where the activity related to a topic being studied. A middle school teacher could (and should) use these activities as well as others as part of a two to three week unit on the topic of problem solving.

PROBLEM SOLVING ACTIVITIES

Problems that Have Historical Connection

There are many topics in mathematics that can and should be introduced or explored in an historical context. Three such topics are discussed in this section.

The Creation of the Coordinate Grid

The development of Cartesian graphing has an interesting story behind it, in which Descartes developed the system while lying on his back in a hospital for a prolonged stay. Attempting in a letter to tell his friend how bored he was, he wrote that he had nothing better to do than watch a fly on the ceiling. Throughout the letter, he periodically wrote about the position of the fly on the tile ceiling. He developed a method for describing the position of the fly, and even speculated about mathematical ways to describe the motion of the fly. Middle school students could be challenged to find a way to name or locate any point on a plane, and after creating their own methods, could learn about the experience of Descartes and his creation of Cartesian graphing.

Summing Numbers form One to Any Number

Carl Fredrich Gauss was only ten years old when he developed a method for summing all numbers from 1 to n, meaning to add numbers, starting with 1, 2, 3, and continuing to any end point. His teacher gave what he considered to a task which could not be accomplished: to add all whole numbers from one to one hundred. The teacher did not expect a correct response, but he did expect his students to work for a lengthy period of time trying, while he (the teacher) could nap. But the teacher hardly had a chance to close his eyes when Carl appeared before his desk, his slate in hand, with the number 5050 written on it. The teacher, not realizing that this was the correct answer, told the student to sit down and work some more, but Carl (who was a troublesome child for his teachers, as many creative children are) proclaimed that his answer was correct. And this is how he obtained the answer:

> Think of the writing the numbers from 1 to 100, but only write the largest and smallest: 1 + 2 + 3 + . . . + 99 + 100. Then write the numbers a second time, under the first list, but this time from largest to smallest:
>
> 1 + 2 + 3 + . . . + 99 + 100
> 100 + 99 + 98 + . . . + 2 + 1
>
> Add the two numbers that are paired vertically, and the sum of 101 results over and over again; it has to, since each time one number increases by one, the other decreases by one.
>
> 1 + 2 + 3 + . . . + 99 + 100
> 100 + 99 + 98 + . . . + 2 + 1
> ________________________________
> 101 + 101 + 101 + . . . + 101 + 101
>
> You end with the same number (101) a hundred times, so multiply 100 times 101. But since each number was used twice, divide the result by two. Thus, the famous summation formula for summing numbers from 1 to n:
> $n (n + 1) / 2$.

Students can be challenged to develop a method of summing numbers from one to any number. They may start with the numbers one through ten, or twenty, where they can try different methods and then easily verify the answer. After some time, it could be suggested that the students find out if there is a way to pair the numbers. I have used this activity with sixth grade students, and some came up with this idea of writing the numbers up to fifty and then writing them backward from fifty to one hundred, like this:

1 + 2 + 3 + . . . + 49 + 50
100 + 99 + 98 + . . . + 52 + 51

This process results in the correct answer, since, rather than dividing the answer by two, they simply divide the list of numbers in half.

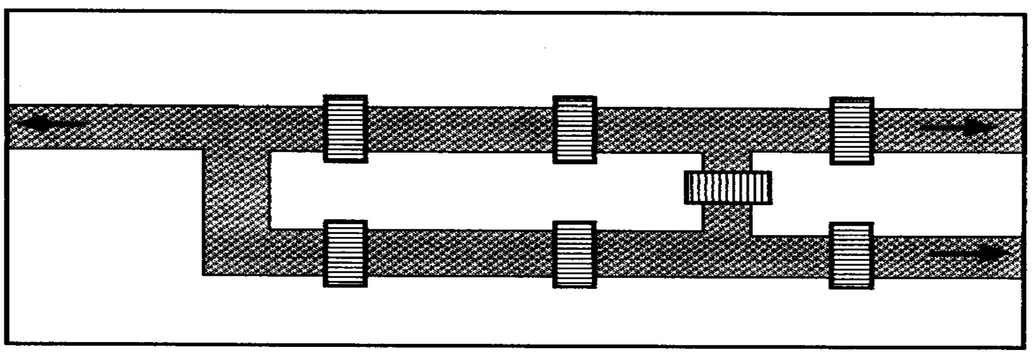
The Seven Bridges of Konigsberg

The problem of the seven bridges of Konigsberg is important because it introduces students to the topology, an area of study that many people either see for the first time in college, or never at all. It is also noteworthy because it is a problem that has no solution, and the idea of proving that a problem is unsolvable is more of a challenge than solving most other problems. the problem is set in the town of Konigsberg, on the Preger River. The town had seven bridges, and the problem was to walk around the town and cross each bridge exactly one, without crossing any bridge twice. Crossing the river in other ways than using a bridge is not allowed.

FIGURE 1: The Seven Bridges of Konigsberg

The Seven Bridges of Konigsberg

The Prussian city of Konigsberg was located on both sides of the Pregel River, and also included two islands. There were seven bridges in the city.



For centuries the citizens of Konigsberg enjoyed taking evening walks around their village. For centuries people wandered if it would be possible to walk around the village and cross each of the seven bridges exactly one time. Try to find a path that will allow you to do this.

* You do not need to end up at the same location you started.

* You may not cross the river in any other way, such as swimming or using a boat.

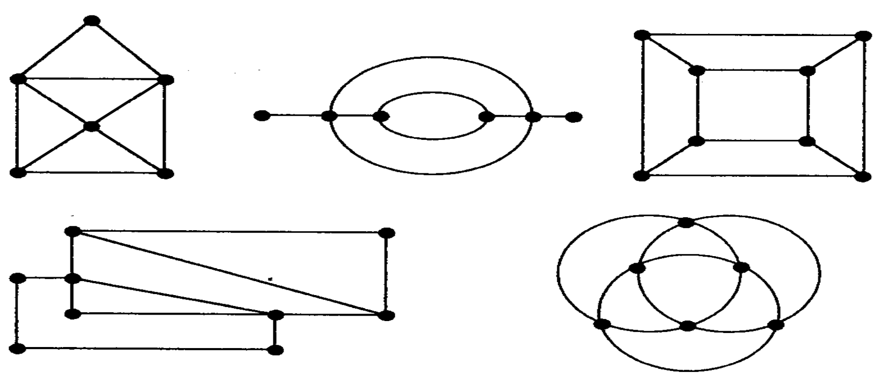
This problem will often engage even the most reluctant learners. They usually do not give easily, and when they do get frustrated, they can be informed that mathematicians worked for more than three hundred years trying to solve this problem. The students may be allowed to take the problem home and continue trying. The next day, or within a few days, the students need to be given a new tool they can use to analyze the problem, and the tool is Network Theory. A network is a set of points (called vertices) connected by lines or curves, called paths. It is fun for students to find out if a network are traceable, that is, if it can be traced without lifting the pen or pencil from the paper, and without going over any path more than once, although a point may be crossed more than. Students also classify each point, or vertex, as even or odd, depending on whether there are an even or odd number of paths connected to the vertex. They then try to develop a rule, based on the number of even and odd vertices, to find out if a network is traceable. The following page could get them started, and they could continue to develop their own networks to gather more data.

FIGURE 2: Networks

NETWORKS

For each network, tell how many even verticies and how many odd vertices it has, whether it can be traced (yes or no), and whether it can be traced starting at any point or only certain points.

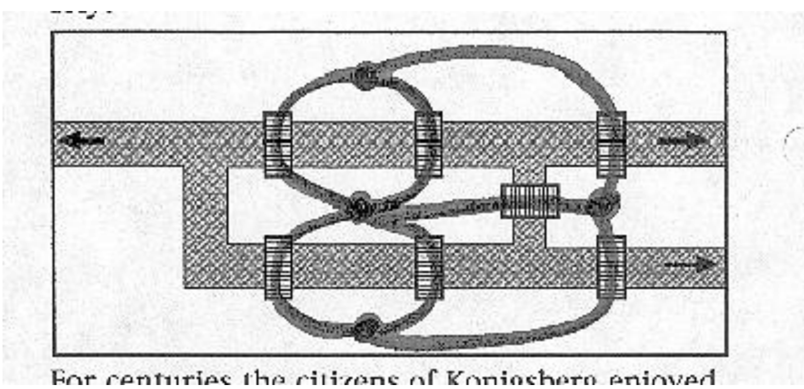
	Even Vertices	Odd Vertices	Traceable (yes or no?)	Any point or certain points?
1				
2				
3				
4				
5				



Through their exploration, students can develop the rule that a network that has all even vertices can be traced starting at any point. A network that has two odd vertices and the rest even can be traced only by starting at one of the odd vertices and ending at the other. A network with more than two odd vertices cannot be traced. The number of even vertices is not relevant to the traceability of a network. This is Leonard Euler's rule for networks.

The Konigsberg bridge problem can be shown to have no solution by representing it as a network, with each piece of land representing a vertex, and each bridge a path. The network will contain four odd vertices and no even, and can not be traced.

FIGURE 3: Solution to the Konigsberg Problem



The Creation of Mathematical Algorithms

If students are to be active participants in the mathematical process, then they should spend time developing the rules and formulas that are used. In the previous section, the problem of summing numbers from one to any number leads to the development of a formula, yet I placed the problem in the previous section because of the historical aspect involved. Because mathematical problem solving can be complex and involve connections to many areas of study, some problems could easily belong to two, or even all three, of the categories I have chosen to discuss. In this section I will discuss the formulas for the calculating the area of a parallelogram and the area of a triangle, using a data gathering approach. There are much practical value in knowing formulas for finding area. I will also examine the problem of finding the area of any shape on a rectangular grid, by discovering "Pick's Rule." This has less practical value, but is an outstanding activity which leads students to gather and organize data, make and test conjectures, share information with each other, and eventually develop a formula for finding the area of any shape on a rectangular dot matrix based on the number of boundary points and the number of interior points (this will be explained shortly).

The Area of Parallelograms and Triangles

This activity uses geoboards and the following activity sheet:

FIGURE 4: Area of Parallelograms and Triangles

▪ AREA OF PARALLELOGRAMS - Complete the chart:

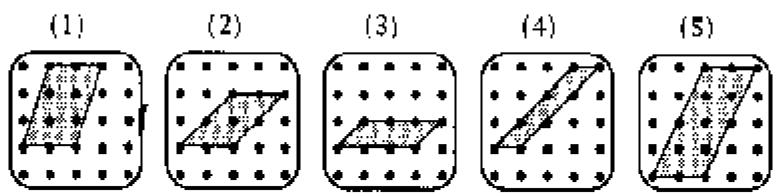


	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5
BASE					
HEIGHT					
AREA					

• AREA OF TRIANGLES - Complete the chart:

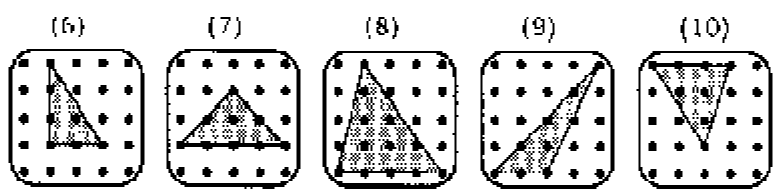


	Figure 6	Figure 7	Figure 8	Figure 9	Figure 10
BASE					
HEIGHT					
AREA					

Students work in groups, using geoboards, to find the area (that is, how many unit squares are inside) of each shaded unit. They may have to divide or rearrange the shapes to obtain the area. They will then fill in the chart and try to obtain a rule that will give the area. The rules are not hard to find, but having the students go through the process of discovering them is much more powerful than simply giving them the formulas. This approach could be used at an easier level, such as finding the area of squares and rectangles, and at a more challenging level for finding the area of trapezoids.

Pick's Rule

Pick's Rule is rather obscure, and the value in the activity in not in the formula itself, but in the process of organizing data and making and testing conjectures in order to find a rule that works. When mathematics is approached in this inductive manner, it becomes similar to the applying the scientific method which involves the creating and testing of hypotheses.

FIGURE 5: Pick's Rule

PICK'S RULE

There is a relationship between the area of any figure drawn on rectangular grid paper and the number of boundary points and interior points. The relationship is known as Pick's Rule.

To discover Pick's Rule, you will gather data and use patterns. Start with shapes that have no interior points, and find a rule that relates are to boundary points.

FIGURE	AREA	BOUNDARY POINTS
A		
B		
C		
D		

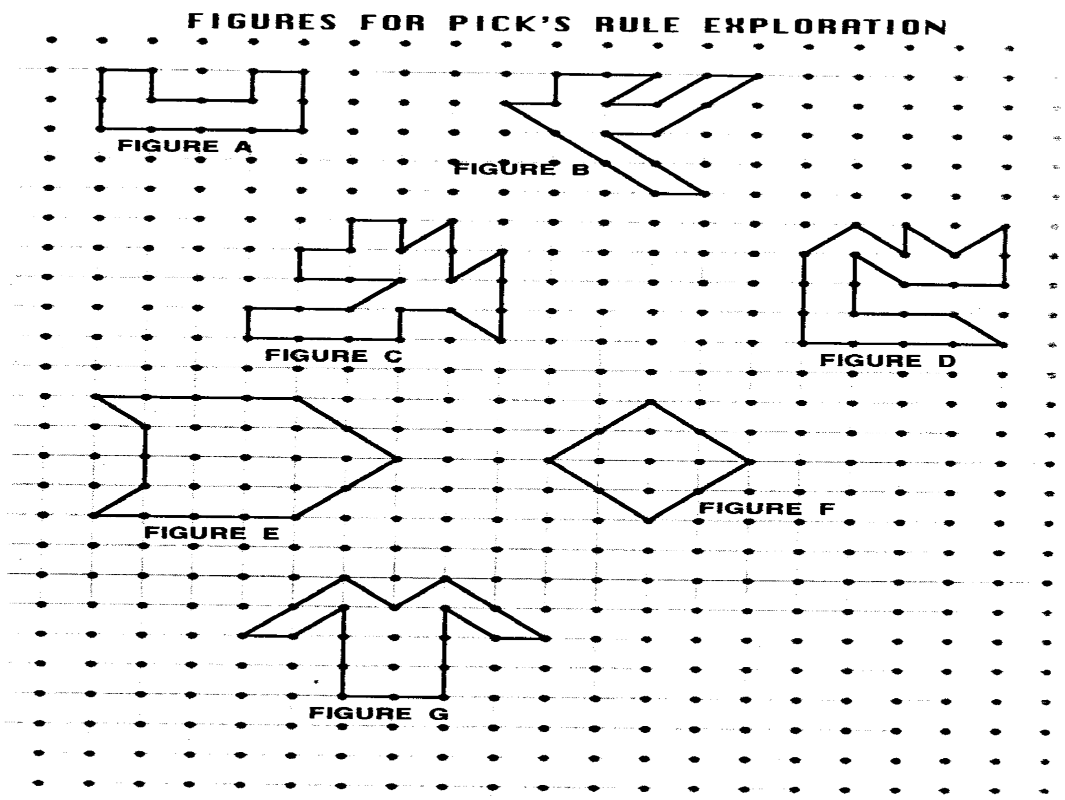
What is the relationship between boundary points and area?

FIGURE	AREA	BOUNDARY POINTS	INTERIOR POINTS
E			
F			
G			

What is the relationship between area, boundary points, and interior points?

The students will complete the charts to and analyze the data to try to obtain the rule. The two tables in the figure above organize the data into two sets: the first obtained from figures with only boundary points (points on the edge of the figure) and the second obtained from figures with both boundary and interior points. Once students find the rule for the figures with only boundary points, they should try to extend it to find the general rule which works for any figure. This activity can be used with geoboards, and the figures below only give the students a starting point. They will have to create and test more figures of their own in order to discover and verify Pick's Rule.

FIGURE 6 - Pick's Rule Data Sheet



Some students may need more guidance from the teacher than others, but all of them should be successful in discovering Pick's Rule:

Area = Boundary points / 2 + 1 + interior points

Solving Non-Routine Problems

Non-routine mathematics problems are those in which the strategy or strategies needed to obtain the solution are not obvious. They are problems which involve more experimentation, problems which will take longer to solve, often involving some trial and error. These problems lend themselves to cooperation among students. As students gain experience in problem-solving, the types of problems which were first non-routine will become routine, and new and more challenging problems will become appropriate. The problems in this section can be used with students who have little or no previous problem-solving experience.

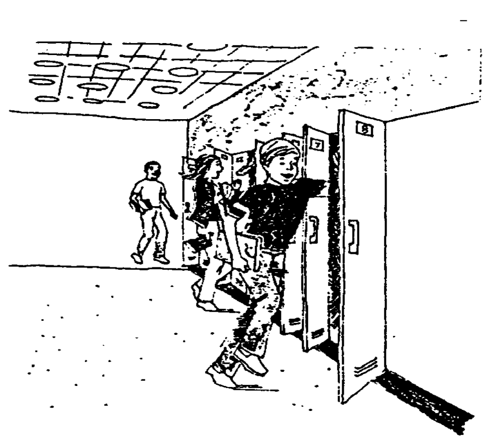
The Locker Problem

The Locker Problem is challenging because it is difficult to know where or how to even start working on the problem. The problem is about a school with one thousand lockers, numbers one through one thousand. The lockers are all closed, and student one comes in and opens all lockers. Then student number two comes in and opens every other locker (the multiples of two). Student three reverses every third locker (the multiple of three), opening those that are closed, closing those that are open. The fourth person does the same to every fourth locker, and so on until one thousand students have come through the school. At this point, which lockers will be open?

FIGURE 7: The Locker Problem

The Locker Problem

At a new junior high school, there are exactly 1000 students and 1000 lockers. The lockers are numbered in order from 1 to 1000. On April Fool's Day the students played the following prank. The first student to enter the building opened every locker. The second student closed every locker that had an even number. The third student *changed* every third locker, closing those that were open and opening those that were closed. The fourth student changed every fourth locker, and so on. After all 1000 students passed through the locker room, which lockers were open?



When I used the problem with middle school students some one would always suggest "acting out the problem" . . . thinking of the joy of spending time in math class running around the hall opening and closing lockers. The suggestion is a good one, but the students can encouraged to simplify the problem. How about twenty lockers and twenty people. Then see which lockers are open, look for a pattern, and predict. In the end, there must be a mathematical reason for certain lockers being open.

Rather than actually going out and opening and closing lockers, students can use posterboard for lockers, numbering from on to twenty, and writing open on one side and closed on the other. Twenty students can walk through and make the changes, and then it will be discovered that lockers one, four, nine, and sixteen are opened. What will be the next locker open? Here is how students often analyze the problem:

1 4 9 16 25
+3 +5 +7 therefore +9

This approach will generate the open lockers, but will be rather time consuming to go up to locker number one thousand. Student should be pushed to look for other patterns. Here is one that is helpful:

1	4	9	16	25
1 X 1	2 X 2	3 X 3	4 X 4	5 X 5

Both methods, if extended, will give the correct answer of thirty-six for the next open locker. But the second method shows that the open lockers will be those with numbers that are perfect squares, that is, numbers that come the result of multiplying a number times itself (squaring a number). Since 31 squared is 961 and 32 squared is 1024, the largest locker that will be opened will be 961, and the answer to this problem is the lockers that will be open are those with perfect square numbers.

Why perfect squares. Students need to explore the underlying reasons that would lead to such a conclusion. To help understand this, let's look at locker number twelve. Which students will open or close this locker? Students one, two, three, and four will, but five will not. Six will, and so will twelve. This could be done for another locker, maybe twenty. The results could list:

Locker number Number:	Opened or Closed by:
12	1 , 2, 3 ,4 ,6, 12
20	1 , 2 , 4 , 5 10 , 20

Those who open or close a certain locker are those who are factors of that locker's number. Students should also discuss when a locker is open or closed. After three people touch change the locker, it will be open. After four, it will be closed. What about is ten people change it? What about fifteen? The students will conclude that an even number of people will leave a locker closed, while an odd number will leave it open. Since every factor has partner (for 12 the 3 and 4 are partners) it would seem at first that all numbers must have an even number of factors. All numbers do, except the perfect squares. Continuing the list above to include sixteen, we see that while the twelve and twenty each have an even number of factors (six), the sixteen has an odd number of factors (five). This is because in a perfect square number, one of the factors is its own partner.

Locker number Number:	Opened or Closed by:	Number of Factors
12	1 , 2, 3 ,4 ,6, 12	6
20	1 , 2 , 4 , 5 10 , 20	6
16	1 , 2 , 4 , 8 , 16	5

Exploring this problem completely will take more than one class period, but because of the many connection to factors and multiple, square numbers, and patterns, it is time well spent.

Sources of Non-Routine Problems

Two excellent sources of non-routine problems are *Creative Problem Solving in School Mathematics* by George Lenchner and *Crossing the River With Dogs, by Ted Herr* and Ken Johnson

These collections give the necessary background so that teachers can fully explore problems with students, and make many connections that would be otherwise overlooked.

CONCLUSION

Students can gain an appreciation for the beauty and intricacy of mathematics by attempting problems in which they must use their own creativity in order to successfully solve the problem. By allowing students to see historical connections and experience the process of creating mathematics themselves, teachers will help students come to understand that mathematics is a living and changing body of knowledge. By requiring students to figure out formulas by gathering data and making hypotheses, teachers will give students the tools needed to move forward in mathematics with confidence. By presenting students with non-routine problems and giving them time to explore, teachers will bring out the creative talents in students that would otherwise never appear in a mathematics class.

ANNOTATED BIBLIOGRAPHY

Teacher Resources

Lenchner, George . *Creative Problem Solving in School Mathematics*
Houghtom Mifflin Company . Boston . 1983

Problems are divided by topic, and most problems have an interesting twist. There are 20 sets of resource problems in the last section of this book.

Polya, George . *How to Solve It: A New Aspect of Mathematical Method*
Princeton University Press, Princeton, NJ 1947

This classic book outlines the steps to be used in mathematical problem solving, and gives some sample problems to help strengthen problem solving abilities.

National Council Of Teachers of Mathematics . *Curriculum and Evaluation Standards* . Reston, VA 1989

The *Standards* are a major part of mathematics education reform in which the teaching and learning of mathematics is more child-centered, focused on meaning rather than memorization, and grounded in problem-solving.

Schoenfeld, Alan H . <u>Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics</u> . from *Handbook of Research on Mathematics Teaching and Learning* . McMillan Publishing Co. New York 1992

This book contains mathematics education research in areas of mathematics teaching, learning from instruction, critical issues, and perspectives.

Student Resources

Barrow, John . *Pi in the Sky: Counting, Thinking, and Being*
Clarendomn Press . Oxford . 1992

An unusual look at what mathematics is, where is comes from, and where it is going.

Dunham, William . *Journey Through Genius: The Great Theorems of Mathematics* . John Wiley and Sons, Inc. 1990

This book is about the great theories of mathematics and the people that developed these theories.

Herr, Ted and Ken Johnson. *Problem Solving Strategies: Crossing the River with Dogs* . Key Curriculum Press, Berkley, CA . 1994

This is a collection of problems and strategies, arranged by topic, that can be used by high school or middle school students. An elective math class could be designed using this book as the main text.

Hoffman, Paul . *Archimedes Revenge: The Joys and Perils of Mathematics* .
Fawcett Crest, New York . 1988

Ann easy to read book which examines mathematics, how some theories were developed and applied, and how mathematics can be both fun and useful.

King, Jerry . *The Art of Mathematics*
Plenum Press . New York and London . 1992

This book looks at the aesthetic beauty of mathematics as seem by the mathematician, but written in a language a common person can understand.

Paulos, John Allen . *A Mathematician Reads the Newspaper* .
Basic Books, New York . 1995

This book, with sections titles like sections of a typical newspaper, analyses situations using mathematics to come to some rather unexpected conclusions.