Symmetry in Wheels

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Introduction

What is so special about symmetry? How is symmetry used in our lives? Are there different forms of symmetry? If there are, how can we tell that symmetry exist? These are just a few questions that could be used to spark an interest in our students. I employ you to let your imagination expand beyond your wildest dreams. This may sound crazy or you may say that you lack a sense of creativity. If so, let your students come up with some ideas. They really like to be included in class lessons.

For instance, without the wheel most of the world's work would stop. Even a mere hubcap of a vehicle has symmetry. "Trains, automobiles, streetcars, farm machines, wagons, and nearly all factory and mine equipment would be useless," exerted from the Compton's Reference Collection. History does not tell us who created the wheel nor when it was invented. But it does tell us that it is the result of long development. To put it gently, symmetry adds a sense of beauty to the world we live in.

Think and Communicate

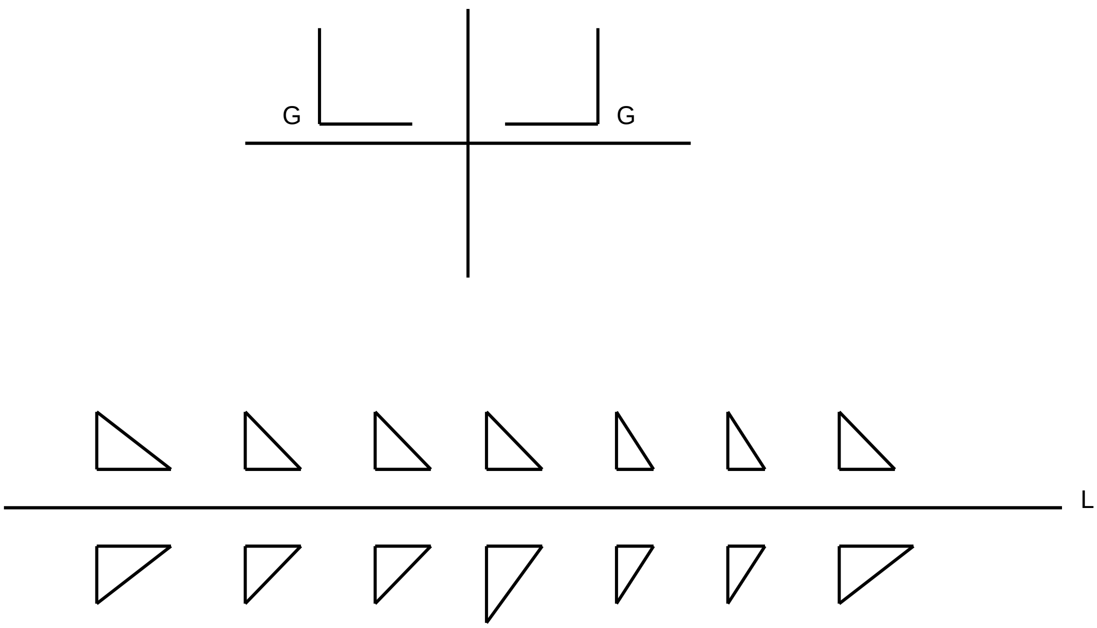
<u>Question</u>: Which figure has only one line of symmetry?

If you said the answer is (C), you are correct. The word, "symmetry often refers to the figure which is an isometry that leaves the figure unchanged. That is, after the isometry is applied to the figure, it looks the same." (Hoffer, 668) In this curriculum unit, I will discuss three kinds of symmetry:

- **<u>Rotational Symmetry</u>**: is a transformation that turns a set of points about one point, the **center of rotation**. The pre-image and image of any point are the same distance from the center of rotation. (Hoffer, 180)
- **<u>Translation Symmetry</u>**: is a transformation that moves all points in a plane a fixed distance in a given direction. (Hoffer, 171)
- **<u>Reflection Symmetry</u>**: is transformation in which each point of a figure has an **image** that is the same distance from the line of reflection as the original point. A point and its image lie on opposite sides of the line of reflection unless the point is on the line of reflection itself. (Hoffer, 391)

To graduate from high school, students will have to complete a long program of study and pass certain examinations. First, in high school, students should take Algebra I, Geometry, 4and Algebra II. It is so vital that students understand that math is a language. Some, if not all, students will gain an understanding of the world around them. Things like pattern, rhythm, form, and sequence **<u>can be</u>** expressed mathematically. I find delight in this and use this in my work to help my students visually express ideas and organize information.

Depending on the type of scheduling for your particular school, the pacing will vary. However, you will be provided with a strong lesson support system for adequate class coverage. In geometry, there are three transformations to consider. First of all, discuss the reflective symmetry, shown in figure A. A figure has reflective symmetry if, when the figure is folded along the line of symmetry the two halves of the figure match (they are mirror images of each other).



Mirror reflectional across central horizontal line. From Washburn1998, pl 46

Figure A

Figure B illustrates our second and third type of symmetry which is rotational symmetry and translation symmetry. A figure has rotational symmetry if it looks the same after a rotation that is less than 360 degrees. The beauty of it all is that the results after one of these movements, or ***transformations***, is called the ***image*** of the original figure. In this curriculum unit, students will look at how transformations were used in the development of an automobile hubcap.

Secondly, challenge your students. I have found, and I'm sure that you would agree, our children are very competitive. Considering the society we live in, competition is evident. Your students can create their own wheel designs. Challenge them to design a figure with specific guidelines: (1) reflective symmetry and rotational symmetry ; and (2) rotational symmetry with an angle of rotation of 90 degrees but no reflective symmetry. (see Figure 1) It is very important to describe these kinds of figures briefly.

There are many objects that also possess symmetry. Textiles, tiles, decorated walls, too name a few, are decorated figures in the plane. (Washburn,43)

Better than using these examples, have your students come up with their own type of symmetry. "Since design is used for balance, the student will venture into a world where a small area of complex shape will balance a large area of simple shape." (Dale Seymour Publications, 2) Realistically, design nor symmetry has no mathematical formula. Moreover, through an abundance of practice and personal experience, they should eventually develop an ability to feel when all the parts of his design are in balance. Henceforth, they are symmetrical.

Finally, the internet provides an array of designs and pictures, colors and value to any lesson. There is an array of lessons that teachers, programs and organizations share with us on symmetry for all grade levels. Students can research all kinds of things. Moreover, the "Symmetry Web is an exploration of the symmetries of geometric figures by means of the World Wide Web" (www.math.okstate.edu/mathdept/symmetry). This project gives you the opportunity to make a selection from a list of symmetries to apply to several geometric figures.

Another web page site demonstrates how an image can have Reflectional Symmetry. "If there exists a line that can be drawn such that the image on one side of the line coincides with the image on the other side of the line then it is said to have reflectional symmetry" (www.geom.umn.edu/~demo5337/s97a/ref-ws.html). In addition, an exploration lab manual, and the Geometry software programs are other provisions that are at your disposal. Frequently, you will find a need to consider a child's special needs and/or learning styles. By having a variety of activities and lessons to choose from, this allows everyone measure of accomplishment.

Moreover, students should think and communicate aloud. Checking key concepts provide a comprehensive review of the concepts. Using technology encourages students to draw their sketches before answering each exercise. With logical reasoning, a student is then responsible for determining whether the polygon's design are similar or not. Communication encourages a student to interact, participate and discuss their findings. There are an abundant of activities we can offer. But with determination, time and structure, the students can be successful.

The Curriculum Unit

In this unit, I am going to discuss the various objectives of reflections, translations, and rotations. The unit opens with the use of reflections to solve real-world problems. Students should learn about translations. They will explore the patterns that involve translation, along with the original figure and its image. Reflections, rotations, and translations are used to move a geometric figure without changing its size or shape. These figures must be related to many real-world applications. Various activities are needed to make this connection.

*A. **OBJECTIVE 1:***

* Draw the reflection of a polygon and a line of reflection for a figure and its image.
* Use reflections to solve real-world problems.

WEEK ONE

PROJECT A: GRIP THE ROAD: (Hoffer, 664)
It is essential to keep the attention of our students. We must take in consideration what interest them. So what interest them more than a football game? ***Answer:*** An automobile to get to the football game. How many kinds of hubcaps are there? Can you identify the make, model, and year of the car for each wheel design? These are just a few questions that can arouse the interest of your students. Using the previous vocabulary, talk with your students about reflective symmetry. Explore with your students these questions:

* Which wheels have a line of symmetry?
* How many lines of symmetry does the wheel have?
* Can you group the wheels by the number of lines of symmetry they have?

There are certain project guidelines that must met for the success of your students. The students will: 1) investigate; 2) set their direction; 3) make a plan; 4) collect and organize their information; 5) carry out their plan; and 6) look back. This is a great opportunity to incorporate collaborative learning. Each group can consist of three (3) to four (4) students. Materials needed would be a compass, straightedge or a ruler, poster board or construction paper and geometry software. Now let's get started.

1) Investigate

* Visit the school parking lot. Ask about the hubcap pattern type and their uses.
* Read about tire manufacture in a sales brochure or in an encyclopedia.

2) Set your Direction

* Will you investigate automobile tires only or truck, wheelchair, and bicycle tires as well.
* Will your poster inform consumers? Explain your answer.

3) Make a plan

* Make a calendar for each day's work. Check in with your group and teacher.
* You'll need posterboard and art materials.

4) Collect and Organize your Information

* Take pictures or make sketches of several hubcaps and patterns.
* Classify them by design and by their vehicle intended.
* Sketch a rough design for your poster. Write a draft of each label, telling how each represented a transformation.

5) Carry Out Your Plan

* Complete your poster according to your sketch.
* Tell which symmetry is more popular: the reflectional or the rotational.

6) Look Back

* What other angle of rotation measures less than 360 degrees rotate a copy of the figure so the copy coincides with the original figure?
* Which wheels have rotational symmetry?
* For each wheel with rotational symmetry, what is the angle of rotation?
* Which wheels have angles of rotation that are factors of 360 degrees? For which wheels is the angles of rotation not a whole number?
* Why do engineers design patterns for hubcaps and rims that have angles of rotation with fractional angle measures?
* Can you group the wheels by the measurement of the angle of rotation?

B. <u>OBJECTIVE 2</u>:

* Describe a translation based on an original figure and its image.
* Describe patterns that involve translation.

WEEK TWO

The goal for this week would include having the students identify the wheels that are not symmetrical. A good synopsis would be if they were pictured without lug nuts. This could be an opportunity to discuss the parts of the wheel that are symmetrical. Ask the students, "How many lug nuts would each wheel need to give the design rotational symmetry?" (Dale Seymour Publications, 4) Its always wise to encourage the students to use critical thinking. Students can find out why an automobile engineering designer often use five lug nuts.

Students who are interested in identifying the make, model, and year of the cars represented by these wheels can work individually or in groups. A list of the cars should be displayed in the classroom or distributed to each student for the child(ren) to match to the wheels. As a Geometry teacher, I realize that real-world experiences are a ***MUST***. Surprisingly, students are enchanted by magazines, newspapers and car brochures. The use of these things in our classrooms will extend the study of the wheel designs to other hubcaps and wheel rims. Talk with your students about the symmetry in the wheels, the elements that are not symmetrical, and shapes within the designs.

Lesson One (Week Two)

<u>Focus</u>: Students will learn how to describe patterns that involve translation.

<u>Drill</u>: (5-minute warm-up) Use Geometric Figures and their Characteristics. Write these problems on the chalkboard.

1. Which figure could contain exactly 2 obtuse angles?(Hint: An obtuse angle is an angle containing more than 90 degrees).

a) A circle

b) A rectangle

c) A trapezoid

d) An obtuse triangle

2. How many lines of symmetry does the following figure exhibit?

a) 1

b) 2

c) 4

d) an infinite amount

3. Which figure has exactly one line of symmetry?

a) a triangle

b) a circle

c) a rectangle

d) a octagon

4. Lakita was given a computer for good grades in school. It was valued at $1099.99. She purchased a computer desk for $79.99, some diskettes for $15.85, a printer for $325, and some printer paper for $24.50. How much did she spend on her purchases (before tax)? (**Hint**: Be careful! One of these amounts doesn't belong in your addition! Read the question carefully.)

a) 235.00

b) 405.75

c) 445.34

d) 515.50

5. Mr. Johnson's delivery truck has a 50 gallon gasoline tank. After filling his tank, he drove 159.8 miles doing his deliveries. After he completed his deliveries, he filled his tank again and found that he had used 7.1 gallons of gas. On the average, how many miles per gallon did he get that day (to the nearest tenth of a gallon)?

a) 15.5

b) 22.5

c) 33.5

d) 42.7

6. Dominique placed one block 4 inches from the wall. The next was 6 inches from the wall and the third block was 9 inches from the wall. Then she placed the fourth block at 11 inches and the fifth at 14 inches. How many inches away from the wall will she place the seventh block?

a) 11 in.

b) 13.4 in.

c) 16 in.

d) 19 in.

<u>New Work</u>: When a picture or a figure is symmetrical, you can draw a line through it that divides it into halves that look perfectly balanced, like the wings of a butterfly. Caleb Crowell and Frank Steele agreed that a figure can be folded along that line so that one of the halves fits perfectly over the other. This dividing line is called a **line of symmetry** or an **axis of symmetry**.

Note: Point Symmetry is another kind of symmetry. A figure has this kind of symmetry if there is a point that is the midpoint of the line segment between any point and its image.

<u>Daily Maintenance</u>: Write these problems on the chalkboard. In their various groups, or individually, have the students express the condition of each problem.

1. Which of the following transformations brings a figure back to its original position?

a) a translation of 2 units up b) a rotation of 360 degrees

c) a reflection about the y-axis d) a reflection about the x-axis

2. (a). Which of the following letters have both line symmetry and point symmetry?

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

(b). Copy the consonants that have both line and point symmetry, and draw all lines of symmetry and centers of symmetry.

3. After a figure was reflected once through the x-axis, its image was identical to the original figure. Explain how this could have happened.

4. A figure is reflected through the y-axis. Then, its image is reflected through the x-axis. Finally, the second image is reflected through the y-axis. Explain how the final image could have been produced with only one transformation.

<u>Explorations Lab Manuel</u>: For additional practice and homework assignments.

<u>Closure Question</u>: How would you describe a translation of a point in coordinate notation? What properties of a figure does a translation preserve?

<u>C. OBJECTIVE 3</u>:

- ∗ Rotate a figure around a center of rotation.
- ∗ Find the coordinates of the vertices of a polygon that has been rotated around the origin.
- ∗ Describe rotations of real-world objects.

WEEK THREE

<u>Drill</u>: Solve each problem. (Irby, 41)

1. Louis's class has 18 boys and 14 girls. What is the ratio of girls to boys?

2. Jim's football team ran 45 passing plays and 27 running plays in its last game. What is the ratio of passing plays to running plays?

3. What is the ratio of even- to odd-numbered sides on a number sides on a number cube with sides numbered from 1 through 6?

4. A weight of 12 grams will stretch a spring 36 centimeters. What weight would stretch the same spring 15 centimeters?

<u>Vocabulary</u>: Have students to familiarize themselves with the following:

1. Rotation: a transformation in which every point moves along a circular path around a fixed point.

2. Center of rotation: the fixed point of a rotation.

3. Angle of rotation: the angle formed by segments drawn from a point and its image to the center of rotation.

Project: Each student should be given construction paper, straightedge, graph paper, and a protractor. Have students plan their model before actually constructing it by making a scale drawings. When making their drawings, students should use reflection and rotational symmetry.

Class Exercises:

1. Describe the transformation of the arrow in black that is needed to produce each image.

From Amsco School Publications, p595

2. For the figure shown, graph the image of the figure after each transformation.
a. Translation down 1 and left 3 b. Reflection through the y-axis
c. Rotation 180 degree. about the origin d. Reflection through the x-axis
e. Rotation 90 degree. clockwise about (5, 1)

Homework Exercises

:

1. If (3, y) is a solution y = 2x - 7, then the value of (y) is:
a) 5 b) 3 c) 2 d) -1

2. The length of a line segment having endpoints (-9, 8) and (-15, 8) is
a) 24 b) 16 c) 6 d) 2

3. A sports jacket that usually costs $90 is on sale for 20% off the regular price. If the rate of sales tax is 5%, what is the total cost of the jacket?

4. The mode of the data 51, 49, 40, 55, 51, 57, 60, 58, 56 is

5. Which figure could have at most one obtuse angle?
a) an obtuse triangle c) a rectangle

b) a trapezoid d) a regular hexagon

6. Which figure can have only one right angle?
   a) square c) trapezoid

   b) parallelogram d) right triangle

7. The motion from the solid figure to the shaded figure is be described as a---

   a) rotation c) inflection

   b) reflection d) translation

8. In the figure, M is the reflected image of P. What is the measure of the angle formed by the line of reflection and the segment connecting points M and P?

   a) 55 degrees c) 90 degrees

   b) 60 degrees d) 180 degrees

9. Find the circumstance of the circle below. Use 3.14 for pi: ________

10. Find the circumference of the circle below. Use 3.14 for pi: ________

EXTRA PRACTICE

Directions: Choose the correct word for each picture. Use reflection, rotation, or translation.

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