# Graphing Linear Equations 

## Reza Khadem

## Introductory Discussion

First of all, here is some information about the school I teach at, and the students I teach. The demographics are from the school Improvement Plan.
"Austin High School is a sixty-two year old school located about two miles southeast of downtown Houston, Texas. The area is made up of low to moderate cost housing units. There are industrial and warehousing facilities in the neighborhood. Austin High School has a student body of 2,400 students. The student population is over $95 \%$ Hispanic, 3\% African American and less than 1\% each of Anglo-American and Asian American. Other student profile data are special ed $12 \%$, at-risk $72 \%$, economically disadvantaged $77 \%$, limited English proficient students $18 \%$. The attendance rate is $90.3 \%$ and the overall dropout rate is $3.1 \%$. The 1999 Spring Exit Level TAAS scores indicate that math went from $65 \%$ to $77 \%$ passing, reading went from $73 \%$ to $82 \%$ passing, and writing from $75 \%$ to $87 \%$ passing. If the 1998 accountability standard was still in place, the scores would have been math $82 \%$, reading $85 \%$, and writing $91 \%$. The Austin staff united together to make TAAS scores their number-one priority. The math and English departments worked as a team to provide efficient and effective instruction to its students. The faculty and staff will continue to work together to make Austin a TEA recognized school."

Here is the TAAS results data from 1995 to 1999. Miraculously, my teaching years at Austin High (1997-1998, 1998-1999) coincides with the years of greatest progress.

|  | 1995 | 1996 | 1997 | 1998 | 1999 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Math | $26 \%$ | $36 \%$ | $32 \%$ | $65 \%$ | $77 \%$ |
| Writing | $60 \%$ | $63 \%$ | $53 \%$ | $75 \%$ | $87 \%$ |
| Reading | $42 \%$ | $55 \%$ | $59 \%$ | $73 \%$ | $82 \%$ |

## Lesson Outlines

The unit on graphing linear equations includes activities that my algebra students did in 1998-1999 school year. These activities worked for my students. I think the lessons are worth trying because they worked in my classroom!

In Lesson One students are asked: " In this activity you create a small design and use mathematics to convert it to numbers. Then, you use the numbers to graph your design
using a graphics calculator. Choose a shape or letter that you can draw using straight lines. Draw your design on a coordinate plane and make sure that the endpoints of th e line segments are on grid intersections. Make sure your design is complex enough to be used in future activities. You will be required to use your design to do a number of mathematical tasks. Make sure to get an "OK" from your teacher before you decide on any design" This activity creates a framework for the rest of the activities in this unit. This framework relates the concepts students learn to a design THEY CREATED.

In Lesson Two students are asked: " In the previous activity you created a design using straight line segments and converted your design to a set of ordered pairs. As you noticed each line segment was created by connecting two points. These line segments have a characteristic (when looked at in their positions in your design) that we c all steepness in English. In algebra there is a concept which is equivalent to steepness in English and that is the SLOPE. In this activity you find the slopes of the line segments from the design you created in the previous activity. Notice that as you move from one point to another on the coordinate plane your position with respect to the origin changes; in other words, there will be HORIZONTAL and/or VERTICAL changes. In algebra we find the slope of the line segment created by moving from one point to another on the coordinate plane by dividing the VERTICAL change by the HORIZONTAL change." This activity builds on the previous understanding of the coordinate system students acquired in lesson one. Building on previous understanding is the natural way of learning new things including mathematical concepts.

In Lesson Three students are asked: " In the previous activities you created a design using straight line segments and converted your design to a set of ordered pairs. Then, you found the slopes of the line segments from the design you had created. In this activity you will find an equation for each of the line segments which slopes you calculated in the previous activity. An (linear) equation is a rule which shows the relation between the vertical and horizontal coordinates of points on a line. For example, $(1,3)$ and $(2,5)$ are on a line whose equation is $y=2 x+1$. In general, the form of the equations you will write will be: $y=m x+b$ where $m$ is the slope and $b$ is a number on the $y$-axis at which the segment (or the extension of the line segment) crosses the y-axis. In this activity students learn how to find an equation of a line given two points. Since the lines they find equation for belong to THEIR OWN DESIGN, the task becomes more meaningful.

In Lesson Four students check their answers with a graphing calculator. In the previous activities you created a design using straight line segments and converted your design to a set of ordered pairs. Next, you found the slopes of the line segments from the design you had created. Then, you found an equation for each of the line segments which slopes you had calculated in the previous activity. In this activity you check your work with a graphing calculator. The calculator will calculate $m$ and $b$ for you! Carefully read the example below and then, fill in the chart that follows. This kind of use of calculator helps students APPRECIATE TECHNOLOGY. I think calculators (or software run on computer) should play an important role in an algebra class. Project one asks students: watch the movie TWISTER. Then, write a short paper about the movie's connection to mathematical concepts you have been studying in class, particularly graphing linear equations. Make sure you include your interpretation of the scene below. This activity
could be modified to one where students watch a clip from the movie twister in class and asked to write about it in class. This activity has an interdisciplinary character; Students get to WRITE about MATHEMATICS. Project two is from the algebra textbook used in Houston Independent School District. It is a great project to be assigned after lesson one. In this project students are asked: make a list of equations you found for the line segments from your design, then trade your list with a classmate. You are supposed to graph your classmate's design using the list of equations. You can use a calculator to help you see where the lines intersect; these intersections are the end points of the line segments.

The activities for these four units are discussed in more detail below.
This unit ends with an Assessment. Since assessment is a part of the process of learning, students should be assessed to help them improve their weaknesses.

## Lesson One -- Objectives

- Create coordinate graphs.
- Graph scatter plots using a graphics calculator.


## Student Resource Materials

- Graph paper and pencil
- Graphics calculator
- Ruler


## Correlation to McDougal Littell ALGEBRA 1

Chapter 2, section 2.4. To practice the skills learned in this activity practice 13 from McDOUGAL practice Bank can be assigned as homework or class work. Also, CONNECTION on page 76 is a good reinforcement practice.

## Lesson One -- Activity

In this activity you create a small design and use mathematics to convert it to numbers. Then, you use the numbers to graph your design using a graphics calculator. Choose a shape or letter that you can draw using straight lines. Draw your design on a coordinate plane and make sure that
the endpoints of the line segments are on grid intersections. Make sure your design is complex enough to be used in future activities. You will be required to use your design to do a number of mathematical tasks. Make sure to get an "OK" from your teacher before you decide on any design.

It of grid paper.

- When drawing your design on a grid paper, fit your design within a 10 by 10 area of the grid if it is possible.
- To enter your numbers into a TI calculator press STAT and ENTER; then enter X values in L 1 and Y values in L 2 and press ZOOM, choose ZOOMSTAT, and ENTER. You should be able to see your entire design. Next, you can experiment with different zoom functions and choose the one that fits your design best.

Lesson Two -- Objective:

- Find the slope of a line


## Textbook correlation:

McDougal Littell Algebra 1, chapter 3 section 3.3; exercise 20 and connection on page 117 are good practice problems to enhance understanding of the concept of slope.

## Student Resource Materials:

- Graph paper and pencil
- Graphics calculator
- Ruler


## Lesson Two -- Activity

In the previous activity you created a design using straight line segments and converted your design to a set of ordered pairs. As you noticed each line segment was created by connecting two points. These line segments have a characteristic (when looked at in their positions in your design) that we call steepness in English. In algebra there is a concept which is equivalent to steepness in English and that is the SLOPE. In this activity you find the slopes of the line segments from the design you created in the previous activity. Notice that as you move from one point to another on the coordinate plane your position with respect to the origin changes; in other words, there will be

HORIZONTAL and/or VERTICAL changes. In algebra we find the slope of the line segment created by moving from one point to another on the coordinate plane by dividing the VERTICAL change by the HORIZONTAL change.

## SLOPE = VERTICAL CHANGE / HORIZONTAL CHANGE

## Example

In this
 design the slope of the line segment from the point $(0,3)$ to the point $(2,4)$ is calculated as:

Slope $=(4-3) /(2-0)=1 / 2$.

Find the slope of all the line segments in your design ( remember division by zero is not allowed and a non zero number divided by zero equals zero):

| FIRST POINT | SECOND POINT | FORMULA SLOPE=RISE / RUN | SLOPE |
| :---: | :---: | :---: | :---: |
| . | . | . | . |
| . | . | . | . |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . |  |
| . | . | . | . |

Lesson Three -- Objective:

- Write linear equations in the form: $\mathbf{y}=\mathbf{m x}+\mathbf{b}$

Textbook correlation:
McDougal Littell Algebra 1, chapter 3 section 3.4 and 3.5; exercise 18 0n page 123 and 26, 27 on page 129.

## Resource Materials:

- Graph paper and pencil
- Graphics calculator
- Ruler


## Lesson Three -- Activity

A: In the previous activities you created a design using straight line segments and converted your design to a set of ordered pairs. Then, you found the slopes of the line segments from the design you had created. In this activity you will find an equation for each of the line segments which slopes you calculated in the previous activity. An (linear) equation is a rule which shows the relation between the vertical and horizontal coordinates of points on a line. For example, $(1,3)$ and $(2,5)$ are on a line whose equation is $y=2 x+1$. In general, the form of the equations you will write will be: $y=m x+b$ where $m$ is the slope and $b$ is a number on the $y$-axis at which the segment (or the extension of the line segment) crosses the y-axis.

## Example

First write the general form of a linear equation: $\mathbf{y}=\mathbf{m x}+\mathbf{b}$. Then, find the slope (remember you calculated this before). Finally, you use the coordinates of one of the points to find $b$.

In this
 design the slope of the line segment from the point $(0,3)$ to the point $(2,4)$ is calculated as:
$m=$ Slope $=(4-3) /(2-0)=1 / 2$. Next, we write the general formula with the calculated slope substituted for $m: y=(1 / 2) x+b$. Then, we substitute the coordinates of $(0,3)$ for $x$ and $y$, and solve for $b$ :
$3=(1 / 2) 0+b$, so we have $3=b$ and the equation is: $y=(1 / 2) x+3$. Notice that if you apply this rule to the $x$ coordinates of both $(0,3)$ and $(2,4)$, you will have the corresponding y coordinates.

Find an equation for all the line segments in your design (how would you deal with undefined and zero slopes?).

| FIRST POINT | SECOND POINT | FORMULA $\mathbf{y}=\mathbf{m x}+\mathbf{b}$ | EQUATION |
| :---: | :---: | :---: | :---: |
| . | . | . |  |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| . | . | . |  |
| . | . | . | . |
| . | . | . |  |
| . | . | . |  |
| . | - | . | . |
| . | . | . | . |
| . | . | . | . |
| - | - | . | . |
| . | . | . | . |

Lesson Four -- Objective:

- Write linear equations in the form: $\mathbf{y}=\mathbf{m x}+\mathbf{b}$ (by the use of $\mathbf{a}$ calculator)


## Textbook correlation:

McDougal Littell Algebra 1, chapter 3 section 3.4 and 3.5; exercise 18 0n page 123 and 26, 27 on page 129.

## Student Resource Materials:

- Graphics calculator
- The work from previous activity
- The movie Twister


## Lesson Four -- Activity

In the previous activities you created a design using straight line segments and converted your design to a set of ordered pairs. Next, you found the slopes of the line segments from the design you had created. Then, you found an equation for each of the line segments which slopes you had calculated in the previous activity. In this activity you check your work with a graphing calculator. The calculator will calculate $m$ and $b$ for you! Carefully read the example below and then, fill in the chart that follows.

## Example

 design I calculated the slope of the line segment from the point $(0,3)$ to the point $(2,4)$ and found an equation for the line which passes through the given points as follows::

Slope $=m=(4-3) /(2-0)=1 / 2$. Next, we write the general formula with the calculated slope substituted for $m: y=(1 / 2) x+b$. Then, we substitute the coordinates of $(0,3)$ for $x$ and $y$, and solve for $b$ : $3=(1 / 2) 0+b$, so we have $3=b$ and the equation is: $y=(1 / 2) x+3$. Notice that if you apply this rule to the $x$ coordinates of both $(0,3)$ and $(2,4)$, you will have the corresponding $y$ coordinates.

Now, I want to check my work with a graphing calculator. Here is how I do it with a TI-82: Press STAT and then ENTER (EDIT should be highlighted before you press

ENTER). Enter 0 and 2 in L1 and 3 and 4 in L2
 , then press STAT, right arrow key, number $5(\operatorname{LinReg}(a x+b))$, and finally ENTER twice. You will


Here $a$ is your $m$ and $b$ is your $b$ and if the points you entered in $L 1$ and $L 2$ lie on a line, $r$ will be equal to 1 .

## Check all the eqautions you found in the previous activity:

| EQUATION YOU <br> CALCULATED | CALCULATOR'S <br> EQUATION | ERRORS (IF <br> ANY) |
| :--- | :--- | :--- |
| - | . | . |
| . |  | . |
|  |  | . |


| . | . |  |
| :---: | :---: | :---: |
| . | . | . |
| . | . |  |
| . | . |  |
| . | . |  |
| . | . |  |
| . | . |  |
| . | . |  |
| . | . |  |
| . | . |  |
| . | . |  |
| . | . | . |
| . | . | . |

## Teachers Resources

| Book Name | Author | Publisher |
| :--- | :--- | :--- |
| Symmetry | Hermann Weyl | Princeton University Press, <br> Princeton, New Jersey |
| Mathematics, the science of <br> patterns | Keith Devlin | Scientific American Library |
| North American Indian <br> Designs | Eva Wilson | British Museum Press |
| Integrating Educational <br> Technology Into Teaching | M.D Roblyer; Jack <br> Edwards; Mary Anne <br> Havriluk | Prentice Hall |
| Algebra 1 textbook | McDougal Littell |  |

