**Simultaneity**

 Are the endogenous variables uncorrelated with the error terms?

Y = α + δ1X + ε (1)

X = θ + β1Y + β2I + u (2)

Substituting for (1):

Y = α + δ1(θ + β1Y + β2I + u) + ε (3)

Y = α + δ1θ + δ1β1Y + δ1β2I + δ1u + ε (4)

Substituting for (2):

X = θ + β1(α + δ1X + ε) + β2I + u (5)

X = θ + β1α + β1δ1X + β2I + β1ε + u (6)

Suppose ε in the equation (1) goes up, Y then goes up, which then increases X.

ε ↑→ Y↑→ X↑, but X is an independent variable and it is now correlated with ε.

∴ Cov(X, ε) $\ne $ 0

Bias is the result.

**Solution:** set all endogenous variables equal to each other explicitly.

For (4):

(Y – δ1β1Y) = (α + δ1θ) + δ1β2I + (δ1u + ε) (7)

Y = $\frac{(α + δ\_{1}θ)}{1– δ\_{1}β\_{1}}+\frac{δ\_{1}β\_{2}}{1– δ\_{1}β\_{1}}I+\frac{(δ\_{1}u + ε)}{1– δ\_{1}β\_{1}}$ (8)

Y = Π11 + Π12 I + V1 (9)

For (6):

(X – β1δ1X) = (θ + β1α) + β2I + (β1ε + u) (10)

X = $\frac{(θ + β\_{1}α)}{1-β\_{1}δ\_{1}}+\frac{β\_{2}}{1-β\_{1}δ\_{1}}I+\frac{(β\_{1}ε + u)}{1-β\_{1}δ\_{1}}$ (11)

X = Π21 + Π22 I + V2 (12)

∴ Y = Π11 + Π12 I + V1 (13)

 X = Π21 + Π22 I + V2 (14)

Simultaneity is now accounted for:

Problem now is identification.