Heterogeneity in Macroeconomics
Empirical Applications

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Intro

- We will study what is called an Huggett/Aiyagari economy.
- Embed the consumption-savings problem into a general equilibrium framework.
- The individual consumer's problem provides a micro-foundation from which we can study aggregate variables and the aggregate implications of policies that affect individuals at the microeconomic level.
- Today we will study stationary equilibria, equilibria in which aggregate variables are time invariant.

Intro

- Consider an economy with a large number of agents facing idiosyncratic earnings shocks.
- How is the interest rate r determined?
- Main complication: the distribution of cash in hand matters! (not just the average).
- We need to understand how to incorporate distributions to the recursive competitive equilibrium

Intro

- The consumer side will look familiar, as it is the same basic model.
- Following Aiyagari (1994), we will add a production sector, much like the representative firm we used before, that demands consumers' savings and labor as inputs to production.
- Lastly we will require that prices are endogenously determined by the interaction of consumers and the representative firm in factor markets.
- In particular, equilibrium in the asset market will endogenously yield an interest rate r such that $\beta(1+r) < 1$ due to precautionary motives.

Model Set up

- **Demographics and Preferences**
  - The economy is populated with a continuum of measure one of infinitely lived, ex-ante identical agents.
  - Preferences are time separable, defined over streams of consumption, given by

$$U(c_0, c_1, ...,) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

    where u satisfies $u' > 0$, $u'' < 0$ and the discount factor $\beta \in (0, 1)$. The expectation is over future sequences of shocks, conditional to the realization at time 0.
  - The individual supplies labor inelastically.

Model Set up

- **Endowment**
  - Each individual has a stochastic endowment of efficiency units of labor $\varepsilon_t \in E \equiv \{\varepsilon^1, \varepsilon^2, ..., \varepsilon^{N-1}, \varepsilon^N\}$.
  - The shocks follow a Markov process with transition probabilities $\pi(\varepsilon', \varepsilon) = Pr(\varepsilon_{t+1} = \varepsilon' | \varepsilon_t = \varepsilon)$ Shocks are iid across individuals.
  - We assume a law of large numbers to hold, so that $\pi(\varepsilon', \varepsilon)$ is also the fraction of agents in the population subject to this particular transition. We assume that the Markov transition is well-behaved, so there is a unique invariant distribution $\Pi^*(\varepsilon)$
  - As a result, the aggregate endowment of efficiency units is constant over time, i.e. there is no aggregate uncertainty.

$$H_t = \sum_{i=1}^{N} \varepsilon_i \Pi^*(\varepsilon_i) \quad, \forall t$$

Model Set up

- **Budget constraint**
  - For individual i at time t, the budget constraint reads

$$c_t + a_{t+1} = (1 + r_t)a_t + w_t\varepsilon_t$$

  - Wealth is held in the form of a one-period risk-free bond whose price is one and whose return, next period, will be $(1 + r_{t+1})$, independently of the individual state (i.e., r_{t+1} does not depend on the realization of ε_{t+1}, i.e. non-contingent).

Model Set up

- **Liquidity constraint**
  - At every t, agents face the borrowing limit

$$a_{t+1} \geq -b$$

    where b is exogenous
  - Alternatively, we could assume agents face the natural borrowing constraint, which is the present value of the lowest possible realization of her future earnings.

Model Set up

- **Technology**
  - The representative competitive firm produces with CRS production function $Y_t = F(K_t, H_t)$ with decreasing marginal returns in both inputs and standard Inada conditions. Physical capital depreciates at rate $\delta \in (0, 1)$.
- **Market Structure**
  - Final good market (consumption and investment goods), labor market, and capital market are all competitive.
- **Aggregate resource constraint**

$$F(K_t, H_t) = C_t + I_t = C_t + K_{t+1} - (1 - \delta)K_t$$

where capital letters represent aggregate variables.

Equilibrium

- **Recursive Competitive Eq.**
  - The stationary equilibrium of this economy requires the distribution of agents across states to be invariant.
  - However, individuals move up and down in the earnings and wealth distribution, so social mobility can be meaningfully defined. Recall that with CM, there is no social mobility: initial rankings persist forever.

Equilibrium

- **Mathematical Preliminaries**
  - The individual is characterized by the pair (a, ε) the individual states.
  - The aggregate state of the economy is the distribution of agents across states, i.e. $\lambda(a, \varepsilon)$.
  - Using the distribution, we can aggregate over individual decisions and compute aggregate variables and prices.

Equilibrium

- **The recursive formulation of the problem**

$$v(a,\varepsilon;\lambda) = \max_{c,a'}\{u(c) + \beta \sum_{\varepsilon' \in E} \pi(\varepsilon',\varepsilon) v(a',\varepsilon';\lambda)\}$$

subject to

$$c + a' = (1 + r(\lambda))a + w(\lambda)\varepsilon$$

$$a' \geq -b$$

Equilibrium

- **Remarks:**
  - Note that in the individual's dynamic program, λ is also a state variable as it is needed to compute market clearing prices.
  - Similarly and for clarity, we have made explicit the dependence of prices from the distribution of agents.
  - However, it is redundant. We have imposed stationarity on the aggregate allocation so the distribution λ is time invariant. It does not change over time, it is not necessary to track its evolution and therefore does not need to be included explicitly as a state variable.

Equilibrium

- **Definition of Stationary RCE**
- A stationary recursive competitive equilibrium is a value function $v : S \to \mathbb{R}$; policy functions for the household $a' : S \to \mathbb{R}$, and $c : S \to \mathbb{R}_+$; policies for the firm H and K; prices r and w; and, a stationary measure $\lambda^* \in \Lambda$ such that:

1. given prices r and w, the policy functions a' and c solve the household's problem and v is the associated value function,
2. given r and w, the firm chooses optimally its capital K and its labor H, i.e.
$$r + \delta = F_K(K, H)$$
$$w = F_H(K, H),$$
3. the labor market clears
$$H = \int_{A \times E} \varepsilon d\lambda^*(a, \varepsilon)$$

- **Definition of Stationary RCE**

4. the asset market clears

$$K = \int_{A \times E} a'(a, \varepsilon) d\lambda^*(a, \varepsilon)$$

5. the goods market clears (redundant by Walras' Law)

$$F(K, H) = \delta K + \int_{A \times E} c(a, \varepsilon) d\lambda^*(a, \varepsilon)$$

6. for all $(\mathcal{A} \times \mathcal{E}) \in \mathcal{B}$, the invariant probability measure satisfies

$$\lambda^*(\mathcal{A} \times \mathcal{E}) = \int_{A \times E} Q((a, \varepsilon), \mathcal{A} \times \mathcal{E}) d\lambda^*(a, \varepsilon)$$

where Q is the transition function defined before. This means, λ^* is a fixed point of the operator T^* such that $\lambda^* = T^*(\lambda)$.

Calibration of the Model

- To solve the model numerically, one needs first to choose values for the parameters. Heres some guidance on how to pick values. Suppose you set the model's period to one year.
- **Technology:** With Cobb-Douglas production function, pick the capital share α to be equal to 1/3. Set the depreciation rate δ to 6%.
- **Preferences:** Typically, we work with CRRA utility. Let γ be the coefficient of relative risk aversion. Acceptable values range between 1 and 5, with values at the low end of the range (say $\gamma = 1$ or $\gamma = 2$) being the most commonly chosen.

Calibration of the Model

- **Discount rate:** As for the discount rate β, it should be chosen so that the aggregate wealth-income ratio replicates the one for the U.S. economy which is around 3. However, this means that the parameter is not calibrated externally, but internally which is computationally painful. So, you could do the following. Imagine that youre in complete markets, then you know that

$$\alpha K^{\alpha-1} H^{1-\alpha} - \delta = \left(\frac{1}{\beta} - 1\right) \implies \alpha \left(\frac{Y}{K}\right) - \delta = \frac{1}{\beta} - 1$$

thus

$$\beta = \frac{1}{1 + \alpha \left(\frac{Y}{K}\right) - \delta} = 0.951$$

In other words, this value for β would give you a K/Y ratio of 3 in complete markets. With incomplete markets the same β gives you a slightly larger capital-output ratio because of the extra precautionary capital accumulation, so one should set β slightly smaller.

Calibration of the Model

- **Borrowing Constraint:** If the natural borrowing constraint is not a good choice for the problem at hand, one could calibrate the borrowing constraint in order to match, say, the fraction of agents with negative wealth which is around 15% in the U.S. economy. The difficulty is that this strategy requires, again, an internal calibration.

Calibration of the Model

- **Labor Income Process:** We want to calibrate the labor endowment shocks to replicate the typical dynamics of individual earnings in the U.S. economy. The best source of data for this is the Panel Study of Income Dynamics (PSID). A decent approximation to U.S. individual earnings dynamics is an $AR(1)$ process like

$$\ln y_t = \rho \ln y_{t-1} + \nu_t \quad \text{with} \quad \nu_t \sim N(0, \sigma_\nu)$$

where $\rho = 0.95$ and $\sigma_\nu = 0.2$. More sophisticated estimates include a transitory component to capture measurement error, as well as less persistent shocks, and a fixed individual component to capture the effect of education, ability, etc

Estimation of Income Processes

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Intro

- Most of the income for most of individuals comes from the labor market.
- Understanding individual income risk is essential to model consumer behavior, to design insurance policy, etc.
- There is a big literature in labor and macro on the estimation of income processes.
- The stochastic process for labor income is a very important ingredient in macro models with heterogeneity.
- The standard assumption is that labor income is the sum of a permanent and a transitory component.

Data

- We want to model earnings dynamics.
- We require the use of panel data.
- Either consumption or income data can be used. However, we normally use income data.
- In the U.S. we normally use the PSID or the SIPP. Sometimes the NLSY can be used.

Data

- This process always starts with a data cleaning procedure.
- Only males?
- Prime age?
- Outliers? Low and high hours? Very low and very high earnings per hour?

Obtain Residual Earnings

- Earnings:

$$\tilde{Y}_{i,j,t} = w_t \exp(f(X_{i,j,t}) + u_{i,j,t})\hat{h} \tag{1}$$

In per hour terms:

$$Y_{i,j,t} = w_t \exp(f(X_{i,j,t}) + u_{i,j,t}) \tag{2}$$

Thus

$$\ln Y_{i,j,t} = y_{i,j,t} = \beta_t + f(X_{i,j,t}) + u_{i,j,t} \tag{3}$$

Structure to the Residuals

- Time invariant model of Storesletten, Telmer and Yaron (2004a)

$$u_{i,j} = \alpha_i + \eta_{i,j} + \epsilon_{i,j} \tag{4}$$

$$\eta_{i,j} = \rho\eta_{i,j-1} + \nu_{i,j} \tag{5}$$

  where

$$\alpha \sim (0, \sigma_\alpha^2), \quad \epsilon \sim (0, \sigma_\epsilon^2), \quad \nu \sim (0, \sigma_\nu^2), \quad var(\eta_{i,-1}) = 0.$$

  and

$$\alpha_i \perp \epsilon_{i,j} \perp \nu_{i,j}, i.i.d$$

- The set of parameters to estimate is then

$$\theta = \{\rho, \sigma_\alpha^2, \sigma_\epsilon^2, \sigma_\nu^2\}$$

Cross-sectional Moments

- Let $m(\theta)_{j,n} = \mathbb{E}[u_{i,j}u_{i,j+n}]$. Then

$$\mathbb{E}[(\alpha_i + \eta_{i,j} + \epsilon_{i,j})(\alpha_i + \eta_{i,j+n} + \epsilon_{i,j+n})] =$$

$$\begin{cases} \sigma_\alpha^2 + \sigma_\epsilon^2 + \sigma_\nu^2 & if \quad j = n = 0 \\ \sigma_\alpha^2 + \rho^n \sigma_\nu^2 & if \quad j = 0, n > 0 \end{cases}$$

Identification through the Autocovariance Function

- Slope:

$$\frac{m_{03} - m_{02}}{m_{02} - m_{01}} = \frac{\sigma_\alpha^2 + \rho^3\sigma_\nu^2 - \sigma_\alpha^2 - \rho^2\sigma_\nu^2}{\sigma_\alpha^2 + \rho^2\sigma_\nu^2 - \sigma_\alpha^2 - \rho\sigma_\nu^2} = \frac{\rho^2(\rho - 1)}{\rho(\rho - 1)} = \rho$$

- Difference:

$$m_{02} - m_{01} = \sigma_\nu^2\rho(\rho - 1)$$

- Covariance

$$m_{01} = \sigma_\alpha^2 + \rho\sigma_\nu^2$$

- Variance

$$m_{00} = \sigma_\alpha^2 + \sigma_\nu^2 + \sigma_\epsilon^2$$

Estimation

- Let $\hat{m}_{j,n}$ be the empirical counterpart of $m_{j,n}$.
- The moment conditions are

$$\mathbb{E}[\lambda_{i,j,n}(\hat{m}_{j,n} - m_{j,n}(\theta))] = 0$$

where

$$\lambda_{i,j,n} = \begin{cases} 1 & if \quad i \quad is \quad present \quad at \quad j \quad and \quad j+n \\ 0 & ow \end{cases}$$

and

$$\hat{m}_{j,n} = \frac{1}{I_{jn}} \sum_{i=1}^{I_{jn}} \hat{u}_{i,j} \hat{u}_{i,j+n}$$

Estimation

- The moments can be expresses as a symmetric matrix

$$\lambda_{i,j,n} = \begin{bmatrix} m_{0,0} & m_{0,1} & \dots & m_{0,n} & \dots & m_{0,J} \\ m_{1,0} & m_{1,1} & & & & m_{1,J} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ m_{n,0} & \dots & \dots & \dots & & m_{n,J} \\ \dots & \dots & \dots & \dots & m_{J-1,J-1} & \\ m_{J,0} & \dots & \dots & m_{J,n} & \dots & m_{J,J} \end{bmatrix}$$

- Let $\bar{M} = vec(\bar{m})$ be the stacked vector of unique observations. Then θ is the solution of

$$\min_{\theta} \left([\hat{\bar{M}} - \bar{M}(\theta)]' W [\hat{\bar{M}} - \bar{M}(\theta)] \right)$$

where W is the weighting matrix.
- Optimal Weighting Matrix, Identity matrix, diagonal of optimal weighting matrix (Blundell, Pistaferri and Preston, 2008).
- Standard Errors as seen in class or bootstrap.

Issues

- Moments in levels (macro) or growth rates (labor)? See Dalym, Hryshko and Manovskii (2017)
- They carry different information. Suppose an individual that appears only once. Observations surrounding missing obs are much lower than the typical ones and more volatile.
- Measurement Error: standard is assumed to be i.i.d across agents and time. Then it is included in the transitory shock.
- Put structure ($MA(q)$ model) to separate transitory shock from measurement error.

Time Varying Parameters

- Storesletten, Telmer, and Yaron (EER, 2001) allow for the conditional variance of the shocks to be different in times of expansions (σ_H^2) versus contractions (σ_L^2).

- $$u_{i,t,j} = \eta_{i,t,j} + \epsilon_{i,t} \tag{6}$$
  $$\eta_{i,t,j} = \rho\eta_{i,t-1,j-1} + \nu_{i,t} \tag{7}$$

  where

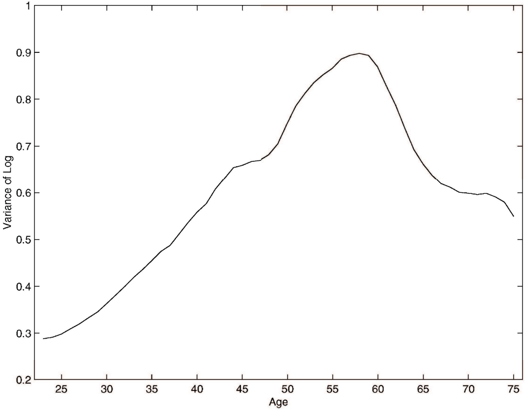
  $$\epsilon_{i,t} \sim Niid(0, \sigma_\epsilon^2), \quad \eta_{i,t} \sim Niid(0, \sigma_\eta^2(Y_t))$$

  and

  $$\sigma_\eta^2(Y_t) = \begin{cases} \sigma_H^2 & if \quad expansion \quad at \quad t \\ \sigma_L^2 & if \quad contraction \quad at \quad t \end{cases}$$

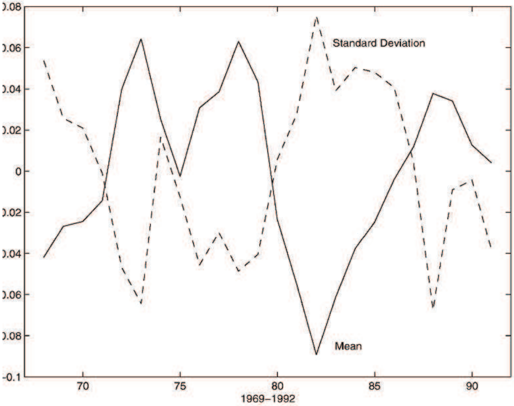
Time Varying Parameters

Over the working years earnings dispersion increases, loosely speaking, linearly ($\rho = 1$).



Time Varying Parameters

Countercyclical heteroskedasticity is a striking feature of the data. The correlation of the detrended mean and the standard deviation is -0.74.



Time Varying Parameters
Identification

- Ignore the transitory shocks. Suppose that there were only three generations: Young, middle aged and old.
- Suppose also that the economy is in an expansion at the current time, but was in a recession during the previous 2 years.
- Suppose that we only observe data dated at the current time, period t.
- The population cross-sectional variances of the idiosyncratic processes, u, for each generation are

$$\begin{aligned} E(u_{i,t,1})^2 &= \sigma_H^2 \\ E(u_{i,t,2})^2 &= \sigma_H^2 + \rho^2\sigma_L^2 \\ E(u_{i,t,3})^2 &= \sigma_H^2 + \rho^2\sigma_L^2 + \rho^4\sigma_L^2 \end{aligned}$$

Time Varying Parameters Estimation

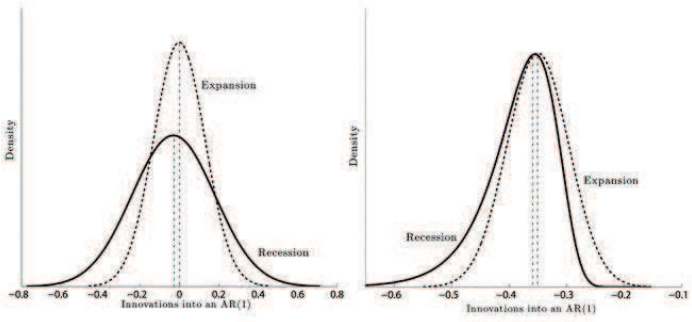
- The method relies on having many obs. on u for each generation. It does not requires to have time-series observations on individual agents.
- The key piece of information we are exploiting is how the cross-sectional variance at date t varies across age cohorts and how this interacts with what is essentially a cohort-specific macroeconomic history which is known at date t.
- Results

$$\begin{aligned}
\rho &= 0.916 \\
\sigma_H^2 &= 0.037 \\
\sigma_L^2 &= 0.181 \\
\sigma_\epsilon^2 &= 0.025
\end{aligned}$$

Guvenen, Ozkan and Song (JPE, 2014)

- Variance of idiosyncratic shocks is not countercyclical.
- Instead, it is the left-skewness of shocks that is strongly countercyclical.
- During recessions, large upward earnings movements become less likely, whereas large drops in earnings become more likely
- Therefore, relative to the earlier literature that argued for increasing variance – which results in some individuals receiving larger positive shocks during recessions – these results are more pessimistic: uncertainty increases in recessions without an increasing chance of upward movements

Guvenen, Ozkan and Song (JPE, 2014)



Long Run Trend in Risk

- Time-varying variances.

$$u_{i,t} = \alpha_i + \eta_{i,t} + \epsilon_{i,t}$$

$$\eta_{i,t} = \rho\eta_{i,t-1} + \nu_{i,t}$$

where

$$\alpha \sim (0, \sigma^2_\alpha), \quad \epsilon \sim (0, \sigma^2_{\epsilon,t}), \quad \nu \sim (0, \sigma^2_{\nu,t}).$$

- Also, Meghir and Pistaferri (Emtca, 2004) allow for a GARCH component in variance terms.

HIP vs. RIP

- *RIP*: Restricted Income Profiles
  Individuals are subject to large and persistent income shocks but have similar life cycle income profiles.

- *HIP*: Heterogeneous Income Profiles-Guvenen (RED, 2009)
  Individuals are subject to income shocks with modest persistence, while facing individual-specific income profiles

HIP vs. RIP

- Guvenen (RED, 2009) revived Lillard and Weiss (1979)

$$u_{i,j} = \alpha_i + \beta_i j + \eta_{i,j} + \epsilon_{i,j} \tag{8}$$

$$\eta_{i,j} = \rho \eta_{i,j-1} + \nu_{i,j} \tag{9}$$

  where α_i and β_i are deterministic individual specific intercept and slope.

- For instance, the source of differences in β can come from returns to human capital accumulation. Early estimates are $0.5 < \rho < 0.7$ and $\sigma_\beta^2 >> 0$

- MaCurdy (1982) cast doubt on these findings. He is not able to reject $\sigma_\beta^2 = 0$. Thus, all the literature evolved assuming RIP and found very large ρ's (> 0.97).

HIP vs. RIP

- Guvenen (RED, 2009)
  - Assuming away the heterogeneity in income growth rates (as is done in the RIP process), when in fact it is present, biases the estimated persistence parameter upward.
  - Intuition: An individual with high (alternatively, low) income growth rate will systematically deviate from the average profile.
  - This fact will then lead the econometrician to interpret this systematic fanning out as the result of persistent positive (or negative) income shocks every period.
  - He provides an example of a simulation in which the persistence parameter is estimated to be about 0.90 if RIP is assumed, instead of the true value of zero.

HIP vs. RIP

- Hryshko (QE, 2012)
  - Use data on idiosyncratic labor income growth from the Panel Study of Income Dynamics.
  - Find that the estimated variance of deterministic income growth is zero, that is, the HIP model can be rejected. The RIP model with a permanent component cannot be rejected.