ECONOMICS 7330, Fall 2022
Bent E. Sørensen

Midterm Exam 2, October 24, 5 questions. All sub-questions carry equal weight.

NOTE: We need to be able to follow your calculations, so just giving a number is not considered a full answer (if we really can't follow your reasoning, it is not even a partial answer).

1. (20%) Assume X and Y are independent exponentially distributed random variables each with mean 2.
Derive the density of $X + Y$. (You do not have to derive the convolution formula.)

2. (20%) Let

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} .$$

a) If X_1 and X_2 are independent, standard N(0,1), normally distributed random variables, what is the distribution of $Y = A\,X$, where $X = (X_1\,,\,X_2)'$. (Describe in words.)
b) Write down the density of Y.

3. (20%) If X is an n-dimensional vector distributed as $N(\mu, \Sigma)$, where Σ has full rank, explain in detail why $(X' - \mu')\,\Sigma^{-1}\,(X - \mu)$ is $\chi^2(n)$ distributed.

4. (20%) Assume that X and Y follow a bivariate Normal distribution with non-zero correlation ρ. Denote the mean, variance of X and Y by μ_X, σ_X^2 and μ_Y, σ_Y^2, respectively.
a) Write down the joint density for X, Y. (You can write it in terms of scalars or in vector/matrix form as long as you clear what is in the vector/matrix. I am asking for numerical values and not just $det|\Sigma|$ or something like that.)
b) Write down the conditional distribution of Y given X.

5. (20%) Consider a sample of N independent log-normally distributed random variables Y_i with finite identical means θ and variance ω^2 and variance of $\log(Y_i) = \sigma^2$.
a) Find the mean and variance of $\Sigma_{i=1}^N Y_i$. (You need to make clear why you can do what you do, don't just state the result.)
Assume you use $\hat{\kappa} = \frac{1}{N}\Sigma_{i=1}^N \log(Y_i)$ to estimate $\log(\theta)$.
b) What is the expected value of $\hat{\kappa}$?
c) What is the RMSE of $\hat{\kappa}$?