ECONOMETRICS I, SPRING 2024

Homework 2. Due Wednesday January 31.

1. Frisch-Waugh with 2 regressors. Assume you regresss

$$y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + error \; .$$

In vector notation:

$$Y = \beta_1 X_1 + \beta_2 X_2 + error \; .$$

Now

$$(X'X) \;=\; \begin{pmatrix} X_1'X_1 & X_1'X_2 \\ X_1'X_2 & X_2'X_2 \end{pmatrix} \; .$$

Find the vector of parameters $\beta = (\beta_1, \beta_2)'$, writing in terms of the inner products (like $X_1'X_2, X_1'Y, X_2'Y$ etc.). It is not hard to invert $X'X$, because the inner products are scalars. If you find it easier, you can assume without loss of generality that the units are chosen such that $X_1'X_1 = 1$ and $X_2'X_2 = 1$.
Now regress

$$X_2 = X_1 \xi + error \; ,$$

and find the fitted value $P_1 X_2 = X_1 * \hat{\xi}$ (remember that $P_1 X_2$ is proportional to X_1) and the residual $M_1 X_2 = X_2 - P_1 X_2$.
Finally, regress

$$Y = (M_1 X_2)\beta_2 \; ,$$

and verify that the $\hat{\beta}_2$ that you get from this second regression is the same as the $\hat{\beta}_2$ from the original regression.

2. Computer question (continuation of homework 1). In Matlab, regress real per capita U.S. data consumption growth on income growth and the interest rate using the posted dataset. (This is the what you did in homework 1.)

a) Regress income growth on the interest rate and take the residual which we can call $M_r Y$.
b) Regress consumption growth on $M_r Y$ and compare the estimated coefficient to the one from the first regression.
c) Regress consumption growth on the interest rate and take the residual which we can call $M_r C$
d) Regress $M_r C$ on $M_r Y$ and compare the estimated coefficient to the one from the first and second regressions.