ECONOMETRICS I, SPRING 2017

Homework 6. Due Wednesday March 8.

1. (15% of a midterm I made at Brown) Assume that you are interested in estimating the model

$$Y_i \;=\; \beta_0 \;+\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \epsilon_i$$

by OLS. Assume that you have 90 observations and that you know that

$$var(X_1) = 3, \;\; var(X_2) = 1, \;\; cov(X_1, X_2) = 0, \;\; cov(X_1, Y) = 5, \;\; cov(X_2, Y) = 4, \;\; var(Y) = 40 \;,$$

where "var" and "cov" denotes the empirical variance and covariances.
a) Find the estimated coefficients b_1 and b_2.
b) Find R^2 and the estimated standard error s^2.
c) Perform a 5% one-sided t-test for the hypothesis $\beta_1 > 1$. (If you could not find s^2 in b) use a value of 2.0).
d) Construct a 95% confidence interval for β_2.

2. (16% of a Brown econometrics final that I posed)
Assume that you have estimated the model

$$Y_i \;=\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \beta_3 X_{3i} \;+\; \epsilon_i$$

by OLS, and that the standard assumptions for OLS - inclusive of normality - hold. We are interested in testing the following restriction

$$\beta_1 + \beta_2 = 1 \qquad \text{and} \qquad \beta_3 = 0$$

(in other words, you test the 2 restrictions on the coefficients simultaneously). Assume that the $X'X$ matrix is given as

$$X'X \;=\; \begin{pmatrix} 15 & 10 & 0 \\ 10 & 20 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

and that your estimated coefficients (using n=18 observations) are

$$\hat{\beta}_1 = .5 \quad \hat{\beta}_2 = .8 \quad \hat{\beta}_3 = 3$$

and that you also found

$$s^2 = 3$$

a) Explain which test you would use to test the restriction and give the formula for the test and state the distribution of the test.
b) Perform the test at a 5% level.

3. Computer question (Monte Carlo). Using Matlab, you will examine how the OLS estimator performs as a function of the sample size and the distribution of the error terms in the regressors. You will use the normal random number generator to generate the error terms, the regressors, and the dependent variable. First, for N=500 draw a vector w of normal errors and construct $X_{1i} = 2 + w_i^2$. Draw another vector v of (standard) normal errors and construct $X_{2i} = 2 + w_i^2 + 4 * \log(v_i^2)$. There is no particular logic to the regressors except I make them correlated and not mean 0.

a) Now set N=20, Also draw a vector e of normal errors with variance 3 and construct $Y = \iota + 2X_1 + 4x_2 + e$ (this means that you only use the first 20 observations of the X's). Estimate $\hat{\beta}$ and construct the t-test for $\beta_1 = 2$ and save it. You do this 100 times and count how often the t-stat exceed the critical value for a 5% two-sided test (using the t-distribution with N-3 degrees of freedom).

Next we will try and figure out if the asymptotic distribution of the t-stat is a good approximation for N=20 and N=200. (You can try other values, when you have the loops set up, you should be able to just change one character to re-run the program).

b) Again set N=20. Draw a vector of standard normal errors and construct $z = e^2 - 1$ (the is a simple way to construct a mean zero error that is not normal). Construct $Y = \iota + 2X_1 + 4X_2 + z$. Estimate $\hat{\beta}_1$ and construct the asymptotic t-test for $\beta_1 = 2$ and save it. You do this 100 times and count how often the t-stat exceed the critical value for a 5% two-sided test (using the normal distribution).

c) Set N=200. Draw a vector of standard normal errors and construct $Z = e^2 - 1$ (the is a simple way to construct a mean zero error that is not normal). Construct $Y = \iota + 2X_1 + 4X_2 + z$. Estimate $\hat{\beta}_1$ and construct the asymptotic t-test for $\beta_1 = 2$ and save it. You do this 100 times and count how often the t-stat exceed the critical value for a 5% two-sided test (using the normal distribution). Does the asymptotic test perform better for the higher value of N?