ECONOMETRICS I, SPRING 2017

Homework 2. Due Wednesday February 8.

1. Let

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

and

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

a) Prove that trace(AB)=trace(BA) .

b) Using this result, prove (for quadratic matrices) that
trace(ABCD)=trace(BCDA)=trace(CDAB)=trace(DABC) .

2. Let

$$C = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

Find the eigenvalues and eigenvectors for C.

3. Show $\sum_i (y_i - \bar{y})^2 = y' M^0 y$ with $M^0 = I_N - \frac{1}{N} \iota_N \iota_N'$.

4. Frisch-Waugh with 2 regressors. Assume you regresss

$$y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + error \ .$$

In vector notation:

$$Y = \beta_1 X_1 + \beta_2 X_2 + error \ .$$

Now

$$(X'X) = \begin{pmatrix} X_1'X_1 & X_1'X_2 \\ X_1'X_2 & X_2'X_2 \end{pmatrix} .$$

Find the vector of parameters $\beta = (\beta_1, \beta_2)'$, writing in terms of the inner products (like $X_1'X_2, X_1'Y, X_2'Y$ etc.). It is not hard to invert $X'X$, because the inner products are scalars.
Now regress

$$X_2 = X_1 \xi + error \ ,$$

and find the fitted value $P_1 X_2 = X_1 * \hat{\xi}$ (notice that $P_1 X_2$ is proportional to X_1 which maybe I should have stressed more in class) and the residual $M_1 X_2 = X_2 - P_1 X_2$.
Finally, regress

$$Y = (M_1 X_2) \beta_2 \ ,$$

and verify that the $\hat{\beta}_2$ that you get from this second regression is the same as the $\hat{\beta}_2$ from the original regression.

4. Computer question (continuation of homework 1). In Matlab, regress real per capita U.S. data consumption growth on income growth and the interest rate using the posted dataset. (This is the what you did in homework 1.)

a) Calculate the vector e or residuals from the regression.
b) Calculate the variance-covariance matrix of the estimated OLS parameters as $\hat{\sigma}^2 * (X'X)^{-1}$, where $\hat{\sigma}^2 = \frac{1}{N-3}e'e$ and print it. (We have not show yet that this is the best estimate of the variance-covariance matrix of the estimated parameters, but trust me that it is for now.)