ECONOMETRICS II, FALL 2023

Homework 7. Due Wednesday October 18.

1. a) Show for 2×2 matrices that $\text{vec}\,(ABC) = (C' \otimes A)\,\text{vec}\,B$. (It is enough to verify it for the first three elements or so of the resulting vector.)

b) Prove that $(A \otimes B)(C \otimes D) = (AC \otimes BD)$ using the formula from part a). This can be done (painfully) by multiplying it all out, but a much more clever way is to prove that $(A \otimes B)(C \otimes D)\,\text{vec}\,(X) = (AC \otimes BD)\,\text{vec}\,(X)$ for arbitrary X. Next hint is to use the formula from a) two times, and first show that $(C \otimes D)\,\text{vec}\,(X) = \text{vec}\,(DXC')$ and then use the formula again.

2. Consider the demand and supply model (or whatever the variables may stand for):

$$y1 = \alpha_1 * y_2 + \alpha_2 x_1 + u_1,$$

and

$$y2 = \alpha_3 * y_1 + \alpha_4 x_4 + \alpha_5 x_5 + u_2.$$

i) Assume you know the Π matrix of the reduced from (this can be estimated consistently), write down and solve the 5 equations for the α's.
2) If instead

$$y2 = \alpha_3 * y_1 + \alpha_6 * x_1 + \alpha_4 x_4 + u_2,$$

show that one cannot solve the equation uniquely for $(\alpha_3, \alpha_6, \alpha_4)$.

3. Use the posted program to estimate a 2SLS estimator for the simultaneous equation model (run the program with, say, 1000 simulations). We removed one line from my program that you have to add. Also, add an OLS estimator of the same equation and show that the results of the OLS estimator are biased.

Change one of the coefficients in the simulation to make the OLS bias worse. Simulate again and show it gets worse.