

**Homework 3. Due Wednesday September 15.**

1. Find the variance and all covariances of an AR(1).
2. Using the posted program posted, simulate and estimate an AR(1)-model.
  - Run the “AR(1) Maximum Likelihood” program.
  - Add to the AR-program an OLS regression of  $x_t$  on a constant and a lag and compare the results (this has to either be done in the same regression or you set a “seed” for the random number generator so that the draws are identical).
  - Try different values of the autoregressive parameter. Does the conclusion change in the autoregressive parameter is very close to unity (like 0.99).
  - The program calculates standard deviations using the inverse Hessian. Add an estimator based on the outer products of the scores and compare the estimated standard errors of the parameters.
3. Calculate the variance and all covariance for an MA(1) model.
4. Using posted program, simulate and estimate MA models.
  - Run the “Moving Average Maximum Likelihood” program, that I posted, 10 times. What is the mean estimated MA-parameter and what is the empirical standard deviation. Compare to the estimated standard deviation.
  - Try and set the initial value for the MA value to 2.0. Run the program 10 times and describe what happens to the estimates. (You can also try other “crazy” values, the more you play around with the computer exercises, the better “feel” for the material you will acquire.)
  - Change the model to an MA(2) process and estimate the parameters (try a few times if it won’t converge).

Note: The purpose of this exercise is partly to highlight that maximum likelihood in principle maximizes the likelihood of a vector of observations and not a sum of marginal likelihoods, even if that is what you most often see. It likely also shows you that initial values matter for convergence of the Newton algorithm. (This is usually not a problem—if it converges to different solutions, that is a problem.)