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ECONOMICS 7330—Probability and Statistics, Fall 2023

Homework 1. Monday August 22. Due Wednesday August 31.

1. Let $A = \{a, b, c, d, e\}$ and $B = \{a, c, e, f\}$.
(a) Find $A \cup B$.
(b) Find $A \cap B$.

2. Describe the sample space S for the following experiments.
(a) Flip a coin.
(b) Roll a six-sided die.
(c) Time waiting for a cab.

3. Prove that $P[A \cup B] \leq P[A] + P[B] - 1$.

4. Prove that $[\bigcup_{i=1}^{\infty} A_i]^C = \bigcap_{i=1}^{\infty} A_i^C$.

5. We observe the price of 2 stocks, stock A and stock B.
The probability that the price of stock A increases is 0.7 and the probability that the price of stock B increases is 0.5. We also know that the probability of the event that the price of either stock A or stock B increases is 0.9.
a) What is the probability that the price of stock A increases at the same time as the price of stock B increases?
b) What is the probability that the price of A increases if you observe that the price of stock B increases?
(State clearly which formulas you used).

6. Let B be an event and $A_1, A_2, ..., A_n$ be n mutually exclusive events. Define $A = \bigcup_{i=1}^{n} A_i$. Also assume $P(A_i) > 0$ and $P(B|A_i) = p$ for all i. Show that $P(B|A)$ is also equal to p. [A Venn diagram might help.]

7. Let $X \sim U[0, 1]$ (uniform distribution). Find the PDF (density) of $Y = X^2$. (Use the formula, or find the CDF first.)