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ECONOMICS 7330 – Probability and Statistics, Fall 2022

Homework 11. Due Monday November 28.

1. Consider the exponential model with density

$$f(t_i) = \frac{1}{\lambda_i} \exp(-t_i/\lambda_i) \, ,$$

where λ_i is the expected waiting time for individual i. We cannot estimate a parameter for each person, but we can estimate the effect of covariates by assuming $\lambda_i = x_i\beta$ where we now have a limited number of parameters as the dimension of β is the number of covariates ("regressors") included.

Find the Score and Hessian (or what Hansen calls "Likelihood Hessian" which is multiplied by –1). (Note, the parameter is now β.)

2. Consider the normal model, with its mean a function of individual-specific covariates (regressors). Here there is just one regressor. The ML-estimator maximizes the log-likelihood (suppressing the π term that does not affect maximum:

$$\Sigma_{i=1}^{N} - 0.5 \log(\sigma^2) - 0.5 \frac{(y_i - \beta x_i)^2}{\sigma^2} \, .$$

The parameter vector now is $\theta' = (\beta, \rho^2)$.

i) Find the ML estimator. (Of β and σ^2. We already found $\hat{\beta}$ in the handout.)
ii) Find the score (a 2-dimensional vector).
iii) Find the Hessian and its expected value.
iv) Show that the information equality holds.