

Final Exam, December 5, 2022—7 questions (100 points). All sub-questions carry equal weight unless noted.

1. (18%) Consider an exponential distribution with mean $\frac{1}{\theta}$.
 - a) What is the Cumulative Density Function (CDF)?
 - b) What is the density function (PDF)?
 - c) Find the mean of X . (You need to derive it, just stating it is not a valid answer. You may want to do integration by parts.)

2. (12%) X is uniformly distributed on the interval from -10 to 2 , and Y is uniformly distributed on the interval from -1 to 1 , and X and Y are independent.
 - 1) Write down the joint CDF for X, Y .
 - 2) What is the probability that $\max(X, Y)$ (largest value of X and Y) is larger than 0 ?

3. (10%) Assume $X \sim \chi^2(9)$.
What is $E(X)$? How did you find that answer?

4. (15%)
 - 1) State the formula for $P(A \cup B)$ in terms of $P(A)$, $P(B)$, $P(A \cap B)$.
 - 2) Prove the formula that you just stated.

5. (15%) Assume that X_1, X_2, \dots are independent, identically distributed random variables with mean μ and finite variance σ^2 . Let $\bar{X}_n = \frac{1}{N} \sum_{i=1}^N X_i$.
Prove that \bar{X}_n converges to μ in probability using Chebyshev's inequality. (The answer will be correct even if you do not exactly remember Chebyshev's inequality, as long as you can explain the important implication of it.)

6. (15%) Assume X_1, X_2, \dots, X_n are independently normally distributed with the mean of $X_i = \mu_i$ and the variance of $X_i = \sigma^2$ for all i .
 - 1) (5%) Write down the formula for the unbiased estimator s^2 of the variance σ^2 .
 - 2) (10%) Show that s^2 is a consistent estimator for σ^2 . (Use the Law of Large Numbers.)

7. (15%) The probability that France wins the (soccer) World Cup is 70 percent while the probability they score at least 2 goals (on average) is 80 percent if they win the Cup. The probability they do not win the Cup and score less than 2 goals is 20 percent.
 - a) What is the probability that France scores 2 or more goals?
 - b) Are the events "France wins the Cup" and "France scores less than 2 goals" independent?
 - c) What is the probability France wins the Cup if the score 2 points on average?