ECONOMICS 7344, Spring 2023
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HOMEWORK 5. Due Monday April 24.

1. (15% of the June 2014 core exam) Consider the case of the 2 agents, 2 periods, 2 states-of-the-world model of Obstfeld-Rogoff Chapter 5.2 (where agents can trade using a full set of Arrow securities). Assume that both agents have utility functions $U(C_0)+E_0U(C_1)$, where $U(C_t) = -\exp(-C_t)$. Assume that the endowment of the first agent is $y_0 = 3, y_1 = 3$ and that the endowment of the second agent in period 0 is $y_0^* = 3$ and in period 1 his or her endowment is $y_1^* = 6$ in the "good state" g. In the "bad state" b the endowment of the second agent is $y_1^* = 0$. Assume that the good state happens with probability 0.5.
a) Derive the formula for the rate of interest as a function of initial endowments and period 1 endowments.
b) Now assume that in period 1 the endowment of the second agent is $y_1^* = 4$ in the "good state" and $y_1^* = 2$ in the "bad state." Assume that the good state happens with probability 0.5. Will the rate of interest go up or down compared to the initial situation? (It is more important that you argue the logic than solving for numbers—you don't have to do numbers at all in the question if you argue clearly.)

2. (20% of the June 2013 core exam) Assume an economy consists of three agents (Jones, Smith, and Cooper) who each maximize a von Neumann-Morgenstern utility function

$$U(C_0) + E_0U(C_1) \ ,$$

where $U(C_t) = 100\,C_t - 0.5C_t^2$. There are two time periods ($t = 0$ and $t = 1$), no storage, and two states of the world, "A" and "B," each having probability 0.5.

The following table gives the state-specific endowments for Jones, Smith, and Cooper:

	Jones		Smith		Cooper	
State of the world:	A	B	A	B	A	B
period 0 endowment	50	50	50	50	50	50
period 1 endowment	25	75	75	25	50	50
period 2 endowment	75	25	25	75	10	10
Probability:	.50	.50	.50	.50	.50	.50

Jones, Smith, and Cooper are the only agents in the world. For the first two sub-questions, ignore period 2.

(a) Assume the agents in period 0 can trade in a bond that matures in period 1 (equivalently, they can borrow from each other in period 0) but not in any other assets. Is the rate of interest positive or negative?

(b) Now, instead, assume the agents have logarithmic, rather than quadratic utility. How will that affect the interest rate (for the correct answer, you need to explain what sign it has and why. You are not asked to find the exact interest rate in this question)? (You likely cannot solve the equations to find interest rates and bond holdings but you can check if the market clears for an interest-rate if zero, and then make your conclusion. However, if you explain the logic, you don't have to write any equations for this sub-question.)

For the remaining questions assume that the agents have access to Arrow-securities for both state A and state B and keep assuming that utility is logarithmic. Consider all three periods now.

(c) Find the term structure of interest rates. (Here, you should find the actual interest rates.)

(d) Why does the rates of interest (from period 0 to period 1, from period 1 to period 2) have the sign you found? (negative, positive, or zero).