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ECONOMICS 7344 – MACROECONOMIC THEORY II, part b, Spring 2023

Homework 4. April 12, due Wednesday April 19.

1. (20% of midterm 2, 2009) Consider the CAPM-model. Assume the world only have two outcomes ("states of the world"). Let X be an asset whose payout PO_X is 100 if "shine" a situation where the (net) market return is 10 percent. "Shine" has probability 0.5. If "rain," PO_X is 200, the net market return is 0, "rain" also has probability 0.5. Assume that the safe rate of interest is 2 percent.
a) What is the expected return (ER_X) to an investment in X?
b) What are the possible returns R_X and their probabilities (in other words, what is the distribution of R_X).

2. Asset A and asset B exist for one period and their returns have identical covariances with the market return. Assume that the market return is higher than the safe rate of interest (you should always make that assumption unless instructed otherwise). The rate return of asset B has a variance that is twice as large as the variance of the rate of return of asset A. Which asset will—if the CAPM holds—have the highest expected rate of return?

3. Assume that an economy is inhabited by identical agents (or a representative agent) with logarithmic utility functions.
a) If the (non-stochastic) growth rate of consumption is 6 percent from period 1 to period 2 and agents have a discount factor of 0.99 (you can approximate the discount rate by 0.01), what is the safe rate of interest (the rate of interest from period 1 to period 2)?
b) (From here on, you need the material on the term-structure of interest rates.)Maintain the assumptions of part (a) but now assume the expected growth rate of consumption from period 2 to period 3 is 8 percent. Using the Euler Equation for the representative agent, find the (period 1) forward rate of interest for period 2 to period 3. (For this question only, you can make the approximation that $E\frac{1}{X} = \frac{1}{EX}$ for a certain variable X.

This approximation is NOT allowed unless it is explicitly said so.)

4. Consider the case where utility is exponential $U(C) = -\exp(-C)$ and C_t is distributed i.i.d. (independently, identically, distributed) $N(\mu, \sigma^2)$. The consumer maximises $\Sigma_{t=0}^{\infty} 0.9^t E_0 U(C_t)$.
a) What is the (period 0) price of a one-period discount bond?
b) What is the price of a two-period discount bond?
Now assume that utility is exponential $U(C) = -\frac{1}{4}\exp(-4C)$.
c) What is the price of the one period discount bond? Explain why it is now different.

Now assume that $C_t = \mu + \alpha C_{t-1} + u_t$, where u_t is i.i.d. $N(0, \sigma_U^2)$.
d) What is the price of a two-period discount bond?
e) What happens to the price of the bond if the variance of u_t doubles? What is the intuition for that?