## Bent E. Sørensen

## ECONOMICS 7344 - MACROECONOMIC THEORY II, part b, Spring 2023

Homework 3. April 5, due Wednesday April 12.

1. $(20 \%$ of Midterm 1, 2016) Assume that a consumer has a utility function $U(C)$ where $U$ is monotonically increasing and strictly concave. Assume that the consumer maximizes

$$
\Sigma_{t=0}^{\infty} \beta^{t} U\left(C_{t}\right)
$$

subject to a flow of known income $y_{t}$ and initial wealth. Also assume that the interest rate is equal to the discount rate.
a) Show that consumption is constant over time.
b) Assume that $y_{0}=10, A_{0}=100$, and

$$
y_{t}=1.1 y_{t-1} .
$$

If the interest rate is $20 \%$ (implying that the discount factor $\beta=1 / 1.2$ ), what is the level of consumption?
2. (This may be hard, but see how far you get. It illustrates very clearly how future credit constraints may affect current consumption.) Two consumers live for 3 periods (periods 1,2 , and 3 ), earns $\$ 100$ in the first period and the distribution of future earnings 190 or 110 with probability 0.5 in each of periods 2 and 3 . The consumers have a quadratic utility function and is-in period 1-allowed to freely borrow and lend at an interest rate that equals their rate of time preference which we for simplicity set to 0 (i.e., the net rate of interest is 0 ). Consumer A is allowed to save in period 2 but not to borrow and the consumer has access to no other assets. Consumer B can borrow and save. Let $C_{t}^{A}, C_{t}^{B}$ be the consumption of consumer A and B , respectively, in period t .
A) Is $E_{1}\left(C_{2}^{B}\right)=E_{1}\left(C_{3}^{B}\right)$ ? (Explain.)
B) Find $C_{1}^{B}$.

Now assume that the agents get a further 10 dollars in period 3, and that this is known in period 1. (This is supposed to capture the logic of an announced tax break.)
C) Is $C_{1}^{i}=E\left(C_{2}^{i}\right)$, for $i=A, B$ ?
D) Find $C_{1}^{A}$ and $C_{1}^{B}$ under the new assumption. For which agent did consumption increase more (or, roughly, which agent has the larger propensity to consume).
3. ( $20 \%$ of Midterm 2, 2016) This question is about the Campbell-Mankiw rule-ofthumb (rot) consumer model.
a) Write down the model and explain the content.
b) Assume that you have time series of data on (aggregate) income and consumption. Let $y_{t}$ be income and $c_{t}$ be consumption. Assume that income is well describe by a stationary $\operatorname{AR}(1)$ in differences and that the covariance between $\Delta y_{t}$ and $\Delta y_{t-1}$ is 0.5 while the variance of $\Delta y_{t}$ is 1.0 . Further assume that when you regress $\Delta c_{t}$ on $\Delta y_{t-1}$ you get a coefficient of 0.4.
Given these numbers, what is the fraction of rot consumers in the Campbell-Mankiw model?

