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ECONOMICS 7344 – MACROECONOMIC THEORY II, Spring 2023

Homework 1, Wednesday March 22. Due March 29.

1. Define the lag polynomials $a(L) = a_0 + a_1 L$ and $b(L) = b_0 + b_1 L + b_2 L^2$. (Notice: in the notes, and in class, it is often assumed $a_0 = 1$ and $b_0 = 1$. This is just for simplification and doesn't matter for any results since you can always re-scale the data and the lag-polynomial such that the first coefficient becomes unity (write a(L) as $a_0\, a'(L)$ where the lag polynomial $a'(L) = 1 + \frac{a_1}{a_0} L$ and similarly for $b(L)$). The constant a_0 will not affect the properties of the lag-polynomial that we care about.)

Assume $a_0 = 1$, $a_1 = -2$, $b_0 = 1$, $b_1 = -.3$, and $b_2 = -.5$.

i) If $x_t = 2, x_{t-1} = -3, x_{t-2} = -2$, $x_{t-3} = 9$, and $x_{t-4} = 9$, what is $a(L)x_t$? and $b(L)x_t$? (This should be a number.)
ii) Find the roots of $a(L)$ and $b(L)$. Are the polynomial invertible (I think I called this stable in class)?
iii) What is $c(L) = a(L)b(L)$? What are the roots of $c(L)$ (hint: this should be easy)?
iv) Find the roots of $c(L)$. Is the AR-model $c(L)x_t = 8 + u_t$ stable?
v) Find the coefficients to the constant, L, and L^2 in the lag-polynomial $a^{-1}(L)$.
Now define the polynomials $d(x) = 1 + .2\, x$ and $e(x) = 1 + .5x^2$.
vi) Find the roots of f(L)=d(L)e(L). Is f(L) invertible?

2. (24% of midterm 1, Spring 2005) Assume that income follows the AR(1) process

$$y_t = 2 + 0.4 y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance 3.
a) Is this time-series process stable?
b) Assume that y_0 is a random variable. For what values of the mean $E(y_0)$ and the variance $\mathrm{var}(y_0)$ will the time series y_t; $t = 0, 1, 2, ...$ be stationary?
c) What is $E_1 y_3$ if $y_1 = 5$ and $y_0 = 2$?

d) Write the infinite Moving Average model that is equivalent to the AR(1) model (*) [assuming that the process now is defined for any integer value of t]. (Half the points are from getting the correct mean term.)

3. Assume that income follows the AR(2) process

$$y_t = 3 + 0.3y_{t-1} + y_{t-2} + e_t$$

where e_t is white noise.
a) Is this time-series process stable?
b) What is $E_{t-1}y_t$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?
c) What is $E_{t-2}y_t$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?What is $E_{t-1}y_t$ if $y_{t-2} = 5$ and $y_{t-3} = 10$?

4. Let

$$x_t \ = \alpha_0 \ + \ u_t \ + \ 0.5 * u_{t-1} + u_{t-2} \ ,$$

where u_t is white noise.
Find the auto-covariances for x_t in terms of σ_u^2 (the variance of u_t).

5. Consider the AR(1) model under the assumption that it is stationary:

$$y_t = \mu + ay_{t-1} + e_t$$

a) Find the mean, variance, and first order autocovariance of y_t.
b) Using the results from a), use the formula for the conditional mean of a normal distribution to find $E(y_t|y_{t-1})$ (in the linear model, you can assume normality for this purpose, the formulas won't depend on that).