**Practice Exam, Econometrics I. This is an exam I have given before.**

Each sub-question in the following carries equal weight.

1. (16%)
Assume that you have estimated the model

$$Y_i \; = \; \beta_1 X_{1i} \; + \; \beta_2 X_{2i} \; + \; \beta_3 X_{3i} \; + \; \epsilon_i$$

by OLS, and that the standard assumptions for OLS - inclusive of normality - hold. We are interested in testing the following restriction

$$\beta_1 + \beta_2 = 1 \qquad \text{and} \qquad \beta_3 = 0$$

(in other words, you test the 2 restrictions on the coefficients simultaneously). Assume that the $X'X$ matrix is given as

$$X'X \; = \; \begin{pmatrix} 15 & 10 & 0 \\ 10 & 20 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

and that your estimated coefficients (using n=18 observations) are

$$\hat{\beta}_1 = .5 \quad \hat{\beta}_2 = .8 \quad \hat{\beta}_3 = 3$$

and that you also found

$$s^2 = 3$$

a) Explain which test you would use to test the restriction and give the formula for the test and state the distribution of the test.
b) Perform the test at a 5% level.

2. (32%)
Assume that you want to estimate the model

$$y_t = \beta_1 x_t + u_t \ ,$$

where the model satisfies the "usual conditions" and var($u_t$)=2.
a) Explain what is meant by the "usual conditions."
Assume that $E(x_t) = 0$ and that var($x_t$)=1 and that $x_t$ and $x_s$ are identically distributed and independent for $s \neq t$.
b) Assume you do not observe $y_t$ but instead observe $w_t = y_t + m_t$, where the $m_t$ variables are iid, with mean 0 and variance 1, and are independent of $x_t$ and $y_t$. Compare (mean, variance) the estimator you get from regressing $w_t$ on $x_t$ to the estimator where you observe $y_t$.

c) Assume you observe $y_t$ but now you do not observe $x_t$ but instead observe $z_t = x_t + m_t$, where the $m_t$ variables are iid, with mean 0 and variance 1, and are independent of $x_t$ and $y_t$. Find the Plim of the estimator you get from regressing $y_t$ on $z_t$ to the estimator where you observe (and regress on) $x_t$.
d) The assumption of c) still holds. Now further assume that you observe a variable $v_t$ such that $v_t$ are iid random variables, with mean 0, $v_t$ is independent of $m_t$ and $u_t$ and $E(v_t v_s) = 0$ for $t \neq s$. The covariance between $v_t$ and $x_t$ is assumed to be .5. Now construct a consistent estimator of $\beta_1$.

3. (16%)
In the standard regression model with $K$ regressors

$$Y = X\beta + u \ ,$$

show that $J$ $(< K)$ times $F$ (where $F$ is the F test statistic) is asymptotically $\chi^2-$distributed where $J$ is the number of restrictions being tested. Assume that the standard assumptions (apart from normality) holds for the model, and that $\lim \frac{X'X}{n} = Q$. You may use directly that plim $s^2 = \sigma^2$, and you may use basic results from class. You may also use the form of the asymptotic distribution of the OLS estimator $b$, without deriving it here.

4. (20%)
a) Derive the formula for the GLS estimator under the usual assumptions.

b) Derive the mean and the variance of the GLS estimator.

5. (16%)
Assume you want to estimate the wage equation

$$W_i = \beta_0 + \beta_1 X_i + u_i \ ,$$

where $W_i$ is the wage of individual $i$ and $X_i$ is the labor market experience of worker $i$.

a) Assume that you suspect that the intercept in the equation is different for men and for women. Explain in detail how you would test this.

b) Assume that you suspect that the slope (the "return to experience") is different for men and for women. Explain in detail how you could test this.