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The effect of interest rates on consumption

Consider the Euler Equation and let us for the moment assume that the interest rate is time-varying and known (i.e., the interest rate is non-stochastic). The Euler Eq. is then

$$u'(c_t) = \frac{1 + r_{t+1}}{1 + \rho} E_t u'(c_{t+1}) \ .$$

For the special case of a CRRA utility function $u(c) = \frac{c^{1-\theta}}{1-\theta}$ (where θ is positive) this implies

$$c_t^{-\theta} = (\frac{1 + r_{t+1}}{1 + \rho}) E_t c_{t+1}^{-\theta}$$

or (using the [innocuous] approximation $\log(1+x) = x$ and the [more serious] approximation $\log E_t c_{t+1}^{-\theta} = E_t \log c_{t+1}^{-\theta}$

$$E_t \Delta \log c_{t+1} = \frac{1}{\theta}(r_{t+1} - \rho) \ .$$

This equation has some clear implications since θ is positive. **The higher the interest rate the higher the expected growth rate of consumption**. This conclusion is likely to hold in more complicated models with uncertainty. (Note that for no uncertainty or quadratic utility we don't need to make the "more serious approximation." In general, be careful about using the approximation $\log E(X) = E \log(X)$, usually it will cost you points. We use it here since this allows us to show a point very simply that holds more generally.)

Notice that θ plays the dual role of capturing aversion towards risk and aversion to unequal consumption in different periods. These are tightly related, of course, unequal consumption across time and unequal consumption across states of the world both are driven by the curvature of the utility function. We will return to that later. Here we basically ignored the issue of risk (differences across states-of-the-world) by doing the log-approximation. If you look at the equation, a high r relative to ρ makes the agent consume relatively more in period $t + 1$. But if the agent "hates" unequal consumption, this effect is smaller, so we divide $r - \rho$ by θ.

Note, however, that it is not obvious if consumption in period t "increase" or "decrease" in response to an unexpected change in the interest rate r_{t+1}. Be careful here: By "increase"

I mean if $\frac{\partial c_t}{\partial r_{t+1}}$ is positive, which is different from the question of whether the growth rate in consumption from t to $t+1$ increase. An increase in the interest rate r_{t+1} may increase or decrease c_t, this depends on the income vs. the substitution effect. We are, however, fairly confident from the above the interest rate increase will increase the growth rate of consumption from period t to $t+1$. This is call the *substitution effect*.

However, Federal Reserve policy makers are often more interested in the effect on the *level* of consumption.

To illustrate the income versus the substitution effect, consider a two-period model, where a consumer at the beginning of period 1 has assets A_1. We assume there is no uncertainty (which is basically what we did above), then the first order condition is

$$c_1^{-\theta} = c_2^{-\theta} \frac{1+r}{1+\rho}$$

so

$$c_1 = c_2 (\frac{1+r}{1+\rho})^{-\frac{1}{\theta}}$$

Assume the agent has income y_2 in period 2 and assets (including first period income) of A_1 in period 1. Then

$$(A_1 - c_1)(1+r) + y_2 = c_2$$

or using the first order condition (Euler equation)

$$(A_1 - c_1)(1+r) + y_2 = c_1 (\frac{1+r}{1+\rho})^{\frac{1}{\theta}} \ .$$

Consider the realistic case of $\theta = 2$ implying that

$$A_1(1+r) + y_2 = c_1(1 + r + (1+r)^{1/2}(1+\rho)^{-1/2})$$

so

$$c_1 = A_1 \frac{(1+\rho)^{1/2}}{1 + (1+r)^{-1/2}} + y_2 * \frac{1}{1 + r + (1+r)^{1/2}(1+\rho)^{-1/2}}$$

Now you can tell that if r increases, the first term increases (the positive term in the denominator decrease), while the second term decrease in r. This is very intuitive: if assets A_1 are very large, the consumer get wealthier when they receive interest on the asset and that increases consumption in both periods. If the consumer has, say, no assets and only future income, then that income gets less valuable when the interest goes up because of discounting.

$$c_2 = c_1 * (1+r)^{1/2}(1+\rho)^{-1/2} = A_1 \frac{(1+r)^{3/2}}{1 + (1+r)^{-1/2}} + y_2 * \frac{(1+r)^{1/2}(1+\rho)^{-1/2}}{1 + r + (1+r)^{1/2}(1+\rho)^{-1/2}}$$

so

$$c_2 = A_1 \frac{(1+r)^{3/2}}{1+(1+r)^{-1/2}} + y_2 \frac{1}{(1+r)^{1/2}(1+\rho)^{1/2}+1}$$

This example illustrates the *income effect*: If initial assets are large enough then when the interest rate increases, c_1 will increase in spite the substitution effect moving consumption to period 2. c_2 will increase more because both income and substitution effects push it up. If assets are negative, the income effect will be negative and c_1 will decrease for sure, while c_2 may increase or decrease depending on whether the negative income effect domination the positive substitution effect when interest rates increase. This is often illustrated in a figure in intermediate macro texts (also in Romer's textbook). You should be able to see this intuitively and be ready to interpret results in term of income and substitution effects when the interest rate goes up or down.