Econometrics 2 (Fall 2020)

Homework 3: Time Series - AR(1)

Due Wednesday on Sept. 16, 2020.

This code estimates an MA(1) model using Maximum Likelihood. The model is:

$$y_t = \beta_0 + y_{t-1} + u_t \ ,$$

with $u_t \sim N(0, \sigma)$.

Set the parameters.

There are 50 simulations with 300 observations per simulation. Set $\beta_0 = 0, \beta_1 = 0.5$ and $\sigma = 2$.

```
clc
clear

global y T AR

T = 300;                          % Number of time periods.
AR = 1;                                                % AR order.
beta0 = 0.5;                                           % Beta 0.
beta1 = 0.5;                                           % Beta 1.
sigma = 2;                                             % Standard deviation.

sim = 50;                          % Number of simulations.
results_mat = zeros(sim,3);                            % Results matrix.
```

Maximum Likelihood Estimation.

In each simulation, generate draw the error terms, U, from the normal distribution and generate the data, Y. Estimate the model using Maximum Likelihood and record the estimates.

```
for s = 1:sim

    u = normrnd(0,sigma,T,1);

    y = zeros(T,1);
    y(1) = beta0/(1-beta1) + u(1)/sqrt(1-beta1^2);

    for j = 2:T
        y(j) = beta0 + beta1*y(j-1) + u(j);
    end

    init = [0 0.99 0.1];

    options = optimset('Display','off','algorithm','quasi-newton');
    [b_mle,~,~,~,~,hess] = fminunc('logl_AR',init,options);
```

```
    results_mat(s,:) = b_mle';

end
```

Display the results of the last simulation.

The estimate and standard error (in parenthesis) of β_0 in the last simulation is:

```
fprintf(' %0.4f\n(%0.4f)\n',b_mle(1),sqrt(hess(1,1)))
```

The estimate and standard error (in parenthesis) of β_1 in the last simulation is:

```
fprintf(' %0.4f\n(%0.4f)\n',b_mle(2),sqrt(hess(2,2)))
```

The estimate and standard error (in parenthesis) of σ in the last simulation is:

```
fprintf(' %0.4f\n(%0.4f)\n',b_mle(3),sqrt(hess(3,3)))
```

Empirical results.

The average and standard deviation (in parenthesis) of β_0 is:

```
fprintf(' %0.4f\n(%0.4f)\n', mean(results_mat(:,1)), std(results_mat(:,1)))
```

The average and standard deviation (in parenthesis) of β_1 is:

```
fprintf(' %0.4f\n(%0.4f)\n', mean(results_mat(:,2)), std(results_mat(:,2)))
```

The average and standard deviation (in parenthesis) of σ is:

```
fprintf(' %0.4f\n(%0.4f)\n', mean(results_mat(:,3)), std(results_mat(:,3)))
```

Plot the estimated coefficients from all simulations.

```
bins = 10;
```

Plot a histogram of β_0.

```
close all
figure(1)
hold on
histogram(results_mat(:,1),bins)
xlabel('$\beta_{0}$','interpreter','LaTex'); ylabel('Frequency')
title('Plot of $\beta_{0}$','interpreter','LaTex')
hold off
```

Plot a histogram of β_1.

```
figure(2)
hold on
```

```
histogram(results_mat(:,2),bins)
xlabel('$\beta_{1}$','interpreter','LaTex'); ylabel('Frequency')
title('Plot of $\beta_{1}$','interpreter','LaTex')
hold off
```

Plot a histogram of σ.

```
figure(3)
hold on
histogram(results_mat(:,3),bins)
xlabel('$\sigma$','interpreter','LaTex'); ylabel('Frequency')
title('Plot of $\sigma$','interpreter','LaTex')
hold off
```