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"Excess Sensitivity" tests of Hall's '78 model

The implication of Hall's 1978 model is that consumption is a martingale; i.e.,

$$E_{t-1}\Delta C_t = 0 \ .$$

The natural way to test this implication (on micro or macro data; but think macro here) is to estimate regression models of the form

$$(*) \quad \Delta C_t = \alpha + \beta X_{t-1} + e_t \ ,$$

where X_{t-1} is a variable that is known at period $t-1$. The coefficient to X_{t-1} (β) is zero according to the model, because $X_{t-1} \in I_{t-1}$ implies that $E\Delta C_t X_{t-1} = 0$. (I, as is typical, do not pay much if any attention to constants, so you may interpret the variables as demeaned. Recall that the OLS estimater of a regression of a Y on and X is the ratio of the empirical coverance to tthe empirical variance of X.) If you want a proof, use the law of iterated expectations:

$$E\Delta C_t X_{t-1} = E\{E_{t-1}\Delta C_t X_{t-1}\} = EX_{t-1}\{E_{t-1}\Delta C_t\} = EX_{t-1}0 = 0\,.$$

Here the second equality sign follows as X_{t-1} is a constant when we condition on I_{t-1}.

So a simple way to test the PIH is to test if a variable *known at* $t-1$ is significant (usually by a standard t-test). In the econometrics jargon, we test if lagged variables have zero coefficients in the equation for consumption growth. (Econometricians will often include more than one lagged variable, and all the coefficient should then be zero, but the interpretation doesn't change if you include several lagged variables at the same time.)

Hall included various variables, including lagged consumption and lagged stock-prices in his tests; and found that only lagged stock prices were significant (i.e., statistically different from 0) in regressions of the form (*). However, later work tend to always find that lagged income is significant in a regression like (*).

In particular, Marjorie Flavin tested if (aggregate) consumption reacts appropriately to income in a much cited paper in the JPE (1981) and use the terminology: "Excess Sensitivity" tests. Her test is somewhat complicated and actually not quite valid either if income is non-stationary (more precisely, if the ARMA model that describes income well is not stable). However, her tests are conceptually more related to the "excess smoothness" tests we covered (although that only became clear to the profession after a while).

Later the jargon "excess sensitivity tests" have become used more loosely to denote tests of whether the coefficients $\beta_1, ..., \beta_k$ are zero in a regression

$$\Delta c_t = \mu + \beta_1 \Delta y_{t-1} + ... + \beta_k \Delta y_{t-k} + u_t \ .$$

Such tests are actually special cases of Hall's 1978 test of the Martingale model for consumption.

The fact that most researchers find a positive (although often small) coefficient to lagged income has become a stylized fact (generally agreed upon) that is referred to as the "excess sensitivity of consumption" (implicitly: to lagged income).

An interpretation of Excess Sensitivity. The fact that the PIH model doesn't fully fit the data, leaves the obvious task of providing a more general model which fits the data. An early much-cited article by Campbell and Mankiw suggested that a fraction λ of consumers are "rule-of-thumb" consumers who consumer current income while a fraction $1 - \lambda$ are PIH consumers. (Note that this material is covered on page 340-342 in Romer, but I don't think Romer provides much intuition for why the technique works.) This means that for "rule-of-thumb" consumers we have

$$\Delta c_t^{rot} \ = \ \Delta y_t^{rot}$$

and for "PIH" consumers we have

$$\Delta c_t^{PIH} \ = \ e_t$$

where (according to the PIH) $E_{t-1} e_t = 0$ (where the meaning of the superscripts is obvious). Then, since $\Delta c_t = \Delta c_t^{rot} + \Delta c_t^{pih}$ and $\Delta y_t^{rot} = \lambda \Delta y_t$, we have

$$(1) \quad \Delta c_t = \lambda \Delta y_t + e_t$$

where $E_{t-1}(e_t) = 0$. The e_t term is the innovation to consumption plus possible measurement error, both independent with past values of the variables. The model (1) can typically not be estimated directly because y_t will likely not be independent of e_t (the Keynesian model directly implies that shocks to consumption (i.e., changes in e_t) will change income, making e_t and y_t correlated). In econometric jargon, y_t is not an exogenous variable.

The standard econometric solution is to use the **instrumental variables estimation** (IV) technique. I will explain it here for a simple case. (Your time is well spent studying this case, since this gives you the intuition for why IV estimation works, an intuition that econometrics classes rarely have time to develop.)

Assume that

$$\Delta y_t = \mu + \alpha \Delta y_{t-1} + u_t \ .$$

This model fits the data for most countries (and states) well. If you estimate this equation by OLS, you get a coefficient $\hat{\alpha}$ which converges to α in large samples. So let us assume for simplicity that we know α. The idea of IV is that instead of regressing Δc_t on Δy_t you regress Δc_t on $\alpha \, \Delta y_{t-1}$. This results in a coefficient

$$\hat{\lambda} = \frac{\alpha \, cov(\Delta c_t, \Delta y_{t-1})}{\alpha^2 \, var(\Delta y_{t-1})} \ .$$

However, according to the model (1),

$$cov(\Delta c_t, \Delta y_{t-1}) = cov(\lambda \Delta y_t, \Delta y_{t-1}) = \lambda cov(\Delta y_t, \Delta y_{t-1}) \ ,$$

since the e_t term has zero covariance with y_{t-1}. The AR(1) model then implies that the covariance of Δy_t with Δy_{t-1} equals $\alpha \, var(\Delta y_{t-1})$. Putting things together we get

$$\hat{\lambda} = \frac{\alpha \, \lambda \, \alpha \, var(\Delta y_{t-1})}{\alpha^2 \, var(\Delta y_{t-1})} = \lambda \ .$$

So the estimated coefficient will be equal to λ when the number of observations is very large (strictly speaking, of course, the estimated coefficient will converge to the true λ).

An **alternative way** of making the point is as follows: Substitute the expression for Δy_t into (1) and get

$$\Delta c_t = \lambda(\mu + \alpha \Delta y_{t-1} + u_t) + e_t \ ,$$

which implies that

$$(2) \quad \Delta c_t = \lambda \mu + \lambda \alpha \Delta y_{t-1} + (\lambda u_t + e_t) \ .$$

In this regression $\lambda\mu$ is just a constant, and $w_t = \lambda u_t + e_t$ is a valid error term since it has expectation 0 and it is uncorrelated with variables dated $t-1$ or earlier. A regression

$$\Delta c_t = \text{constant} + \beta \Delta y_{t-1} + w_t \ ,$$

will therefore (for the number of observations large) result in an estimated $\beta-$ coefficient that equals $\lambda\alpha$ because this is a valid regression with the error term independent of the regressor. We could find λ from estimating β in equation (2) and solving for $\lambda = \beta/\alpha$. The reason that instrumentals variables estimation typically doesn't follow this approach is that it only works in the one-regressor case: if, for example,

$$\Delta y_t = \mu + \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + u_t \ ,$$

you can still use the IV strategy by constructing $y_t^{iv} = (\alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2})$ and running the regression

$$\Delta c_t = \lambda \, y_t^{iv} \ .$$

The constructed variable y_t^{iv} is called an instrument for y_t.

As a final technical note, be aware that the IV method is very shaky is α is near 0. Implicitly you divide out by α and if α is estimated, rather than known, your results can be very noisy indeed—think of a case where $\alpha = .001$ then a little noise will give $\hat{\alpha} = -.001$ and dividing by $\hat{\alpha}$ rather than α will give large errors. Or, you can observe that the method breaks down if α is equal to zero, and your intuition should be that results are "crazy" if α is very close to zero. That kind of intuition usually holds, if results are not valid for a particular value of a parameter, they are badly behaved in the vicinity of that value. In the particular case considered here, the rigorous econometric literature that demonstrates this point can be found under the headings of "weak instruments."

Campbell and Mankiw found that about 40-50% if consumers are rule-of-thumb consumers, although other papers may find somewhat different ratios. However, the model is quite simplistic and shouldn't be taken literally in the view of many economists. (The main problem is, of course, that the model doesn't provide a theoretical foundation for why such a large fraction of individuals would follow such a mechanical rule.) Nevertheless, the technique is clever (may come in handy in your research) and the Campbell-Mankiw model provides are very good demonstration of the deviations from "perfect PIH behavior" although further research should explore more explicit explanations for why it seems that a large fraction of consumers consume their current income. (This is exactly what is being done at the frontier of research in consumption, most of this research still departs from the "classic models" covered in this class, which is one major reason that we cover these. The second major reason is that you need to learn the tools used.)

Shea's test (Romer 3rd ed, p. 359-360.) Shea's test is not as well known, but I want you to know this test as it exemplifies an alternate approach to testing that is often more convincing and, I think, becoming more common. Shea's test uses micro data, which for sure is becoming the dominant source of data for testing the PIH and its many extensions. Shea found a fairly large sample of households with clear information about future consumption growth (union contracts). He then simply tested if consumption increased when the **income actually arrived** or at the time that that households **learned that income would increase in the future** advance. He found that consumption to a large extent increase when the money arrived, not when the information arrived as predicted by the Hall PIH. Tests like this are not as "clever" as the Campbell-Mankiw test, but because they are simple, they are more robust and to a very large extent the (empirical) research frontier moves ahead through finding unique informative data sets, rather than through clever ideas.