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The equity premium puzzle

Prescott and Mehra pointed out the following.

Consider the Euler equation for a CRRA utility function:

$$C_{t-1}^{-\rho} = E_{t-1}\frac{1}{1+\theta}C_t^{-\rho}(1+s_t)\,,$$

where s is the return to the stock market. Or

$$1 = E_{t-1}\frac{1}{1+\theta}(\frac{C_t}{C_{t-1}})^{-\rho}(1+s_t)\,.$$

Now (for brevity) drop the time index and use g for the growth-rate of consumption and get

$$1+\theta = E(1+g)^{-\rho}(1+s)\,.$$

The Euler equation also holds for the safe interest rate r, so

Now do a second order Taylor around $g = 0, r = 0$ (make sure that this is still "inside the expectation" because we are interested in risk. The derivatives are

1. first order wrt s : $(1+g)^{-\rho}$
2. first order wrt g : $-\rho(1+g)^{-\rho-1}(1+s)$
3. second order wrt s: 0
4. second order wrt g : $\rho(\rho+1)(1+g)^{-\rho-1}(1+s)$
5. second order cross: $-\rho(1+g)^{-\rho-1}(1+s)$

Evaluated at $0, 0$:

1. first order wrt s: 1
2. first order wrt g: $-\rho$

3. second order wrt s: 0

4. second order wrt g : $\rho(\rho+1)$

5. second order cross: $-\rho$

So the second order Taylor is

$$1+\theta = E(1+s-\rho g+0.5\rho(\rho+1)g^2-\rho g\, s)\,.$$

Using that $cov(x,y) = Exy - ExEy$ (and this also holds for the variance) we rewrite in terms of the easier-to-interpret covariances

$$1+\theta = 1+Es-\rho Eg+0.5\rho(\rho+1)(var(g)+(Eg)^2)-\rho(cov(g\, s)+EgEs)\,.$$

We actually do not want a second order approximation, so let us drop squares and products of Eg and Es and get

$$1+\theta = 1+Es-\rho Eg+0.5\rho(\rho+1)var(g)-\rho cov(g\, s)\,.$$

(Wait! Why do we do a second order expansion on the drop the second order terms? This is a bit of a trap: the covariances and variances are first-order objects, so be very careful with expansions within an expectation). Now let us observe that as we used the Euler equation, this holds for any asset, including the safe asset with safe return r. So

$$1+\theta = 1+r-\rho Eg+0.5\rho(\rho+1)var(g)\,.$$

Now subtract the equation for the safe asset from the equation for the stock and simplify we have

$$Es-r = \rho cov(g\, s)\,.$$

Prescott and Mehra found $\overline{s}-\overline{r}$ =0.06, using a broad stock-market index and short-term government bonds for a very long time (1890-1979). THey found a covariance of consumption growth with the market of 0.0024. This implies that for the equation to hold, we need a coefficient of risk aversion of 30, which is very large. (Prescott and Mehra pointed out that this implies that people would rather lose 17% of their income for sure, than having a 50-50 percent change of losing 20%. (Note, that you would typically have to translate a result

on risk aversion into something like this for interpretation.) This is not reasonable in the opinion of most economists. Updating the data makes little difference. This finding is known as the Equity Premium Puzzle. Since the Prescott-Mehra paper many "solution to the equity premium puzzle" articles have been published and as many that refutes the solutions.