ECONOMETRICS 1, Spring 2017
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Midterm Exam - March 22, 2017

Each sub-question in the following carries equal weight.

1. (20%) Assume that you have estimated the model

$$Y_i \;=\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \beta_3 X_{3i} \;+\; \epsilon_i$$

by OLS, and that the standard assumptions for OLS - inclusive of normality - hold. Assume that you used 10 observations. Let β be the column vector $(\beta_1, \beta_2, \beta_3)'$. We are interested in testing the following restriction:

$$R\beta = 1 \;,$$

where $R = (0, 1, 1)$. Assume that the inverse of the $X'X$ matrix is given as

$$(X'X)^{-1} \;=\; \begin{pmatrix} .2 & .1 & .0 \\ .1 & .2 & .0 \\ .0 & .0 & .001 \end{pmatrix} .$$

and that your estimated coefficients are

$$\hat{\beta}_1 = .5 \quad \hat{\beta}_2 = .6 \quad \hat{\beta}_3 = 3$$

and that you also found the estimated variance of the error term to be

$$\hat{\sigma}^2 = .2$$

a) Explain in detail which test you would use to test the restriction and give the formulas.
b) Perform the test at a 5% level.

2. (15%) Assume that you have estimated the model

$$Y_i \;=\; X_i \beta \;+\; \epsilon_i$$

by OLS, and that the standard assumptions for OLS - inclusive of normality - hold. Assume that you have 5 observations of (X_i, Y_i) where the X matrix takes the values

$$X \;=\; \begin{pmatrix} 1 & 1 \\ 1 & -2 \\ 1 & .3 \\ 1 & -2 \\ 1 & 1 \end{pmatrix} .$$

Also assume that you find the residual vector $e = (1, -2, 0, 2, -1)'$, and that you estimate $\hat{\beta} = (2, 3)'$. If you construct

$$Z \; = \; 3X\hat{\beta} + W \; ,$$

where $W = (3, 3, 3, 3, 3)$, what is then the projection $P_Z e$ of the residual vector e on Z?

3. (15%) Assume that you want to estimate the following model using quarterly data for 10 years:

$$y_t = \beta_0 \; + \; \sum_{k=1}^{3} \beta_k D_{kt} \; + \; \beta_4 x_t \; + \; \epsilon_t \; ,$$

where all the "OLS-assumptions" - including normality of ϵ_t - hold. The regressors D_{kt} are quarterly dummy variables, such that

$$\begin{aligned} D_{1t} &= 1 \text{ in the 2nd quarter ; 0 otherwise} \\ D_{2t} &= 1 \text{ in the 3rd quarter ; 0 otherwise} \\ D_{3t} &= 1 \text{ in the 4th quarter ; 0 otherwise} \end{aligned}$$

Now assume that $\bar{y} = 5$ and if we let $\bar{y}_j$; $j = 2, 3, 4$ denote the average of the y-values in the kth quarter, assume that

$$\begin{aligned} \bar{y}_2 &= 4 \; , \\ \bar{y}_3 &= 2 \; , \\ \bar{y}_4 &= 0 \; . \end{aligned}$$

Also assume that $\bar{x} = 0$ and that x_t is orthogonal to D_k ; $\; k = 1, 2, 3$.

Based on the given information, find the values of the OLS-estimates $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$ and $\hat{\beta}_3$.

4. (15%) Assume that you want to estimate the model

$$Y_i \; = \; \beta_1 X_{1i} \; + \; \beta_2 X_{2i} \; + \; \epsilon_i \; ,$$

where X_1 and X_2 are orthogonal regressors.

Assume that *all* the assumptions for OLS to be efficient holds, but you accidentally estimate the model

$$Y_i \; = \; \beta_1 X_1 \; + \; \beta_2 X_2 \; + \; \beta_3 X_3 \; + \; \epsilon_i \; .$$

Assume that a regression of X_3 on X_1 gives an R^2 of 0, whereas a regression of X_3 on X_2 gives an R^2 of .999.

a) This inclusion of X_3 creates a problem - what is that called and how does it affect the estimated

parameters (explain how it affects the properties of the OLS estimator of both β_1 and β_2).
b) What is the expected value of the OLS estimators $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$?

5. Computer question (35%). Read the Matlab code below and answer the questions in the code.

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Econometrics 1
% Spring 2017
% Midterm 1
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;
clc;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% This code estimates the model
%
%                   y = beta0 + beta1*X1 + beta2*X2 + e
%
% using OLS and calculates other things.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Generate the data.

n = 500;                        % Sample size

X1 = randn(n,1);                % X1
X2 = randn(n,1);                % X2

X = [ones(n,1) X1 X2];          % X matrix with constant

beta = [1; 3; 2];               % True coefficients

u = randn(n,1);                 % Standard normal disturbances

y = X*beta + u;                 % Observed values of y
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% Estimate the coefficents using OLS.

b = inv(X'*X)*X'*y;             % OLS estimates

% Compute the standard errors.

k = size(beta, 1);              % Number of coefficients

yhat = X*b;                     % Predicted values of Y
uhat = y - yhat;                % Residuals

s2 = (uhat'*uhat)/(n-k);        % S Squared

vc = s2*inv(X'*X);              % Variance-Covariance Matrix

se = [sqrt(vc(1,1));...
      sqrt(vc(2,2)); ...        % Standard Errors
      sqrt(vc(3,3))];

% Compute the t-statistics.

t = b./se;                      % t-statistics
t = abs(t);                     % Absolute value of t-statistics

disp(' ')
disp('Model: y = beta0 + beta1*X1 + beta2*X2 + e')
disp(' ')

disp('Regression Results')
disp(' ')

disp('   Estimates    SE     |t-stat|')
disp([b se t])
disp('Note: OLS estimates are b0, b1 and b2 in that order.')
disp(' ')

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%
% Question 1: Complete the code below by writing what Matlab should set R
% and varm equal to.
%
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% Compute the F-statistic to test the joint hypotheses that beta1 = 0 and
% beta2 = 0.

R = XXXXXXXXXXXXXX;             % Missing code
q = [0; 0];                     % Hypothesized values

J = size(R,1);                  % Number of restrictions

h = R*b - q;                    % Sample discrepancy
varm = XXXXXXXXXXXXXX;          % Missing code

F = (h'*inv(varm)*h)/J;         % F-statistic

disp('F Test')
disp('H0: beta1 = 0 and beta2 = 0')
disp(' ')

disp('     F-stat    ')
disp([F])
disp(' ')

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%
% Question 2: Complete the code below by filling in the comments,
% identifying what AA, BB, CC and DD compute.
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i = ones(n,1);                  % Iota
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DD = eye(n) - (1/n)*i*i';       % This computes XXXXXXXXXXXXXX.

AA = b'*X'*DD*X*b;              % This computes XXXXXXXXXXXXXX.
BB = y'*DD*y;                   % This computes XXXXXXXXXXXXXX.

CC = AA/BB;                     % This computes XXXXXXXXXXXXXX.

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