ECONOMETRICS 1, Spring 2017
Bent E. Sørensen

Final Exam - May 1, 2017

Each sub-question in the following carries equal weight.

1. (20%)
a) Verify the information matrix equality for the exponential likelihood function with density $\theta e^{-\theta x}$. (For the exponential distriution, $EX = 1/\theta$ and $EX^2 = 1/\theta^2$.)

b) Explain how to estimate the information matrix consistently using only first derivatives of the likelihood function.

2. (30%)
Consider the standard linear regression model

$$Y \; = \; X\,\beta + u\,.$$

a) Explain what instrumental variables (IV) estimation is and when it is used (i.e., when IV estimators are consistent but the OLS estimator is not).
b) Assuming the variance of u is $\sigma^2 I$. Derive the variance of the IV estimator, assuming the instrument vector is Z (you can assume the number of instruments are equal to the number of regressors).
c) Based on the expression you find for the variance, explain when the IV estimator is likely to have low variance. (I want you comment on the components of the expression you just derived.)
d) Now assume that the variance of u is a known symmetric positive definite matrix Ω. If $\tilde{X} = P_Z\,X$ where $P_Z = Z(Z'Z)^{-1}Z'$ is the projection matrix for Z, then suggest an estimator of β that is more efficient than the usual IV-estimator $\hat{\beta}^{IV} = (\tilde{X}\tilde{X})^{-1}\tilde{X}'Y$. (Write down the formula for the estimator.)

3. (10%) In the simple linear regression model

$$y_i \; = \; \beta_0 + \beta_1 z_i + \epsilon_i \; ; \quad i = 1, ..., n \; ,$$

show that $R^2 = r_{yz}^2$, where r_{yz} is the simple correlation between y and z.

4. (10%) Explain (give the formulas) how the White heteroskedasticity-consistent variance estimator is calculated.

5. (10%) a) Is the AR(2) process

$$(1 - 0.5L - 0.5L^2)x_t \;=\; u_t$$

a stable process?
b) If the variance of X_1 is σ_X^2, what is the second order auto-covariance $E(X_3\,X_1)$?

6. (20%) Matlab question. You want to estimate the model

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u\ .$$

You have data for y, X_1, X_2 and another variable W for 10000 individuals. W is an instrument for X_1.

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
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% To do: Fill in the missing code.
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% Estimate the coefficients.

n = size(y,1);                              % Sample size.

X = [ones(n,1) X_1 X_2];                    % X matrix.

Z = XXXXXXXXXXXXXXXXXXXXXXX                   % Z matrix.

b_est_IV = XXXXXXXXXXXXXXXXX;                  % IV-Estimates of beta.

u = XXXXXXXXXXXXXXXXX;                      % Residuals.

var_est = XXXXXXXXXXXXXXXXX;                % Variance-Covariance Matrix.

SE = XXXXXXXXXXXXXXXXX;                     % Standard errors.

t = XXXXXXXXXXXXXXXXX;                      % t-statistics.
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