ECONOMETRICS II, Fall 2022

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Midterm 1—September 26, 2022.

Each sub-question in the following carries equal weight except when otherwise noted.

1. (10%) Consider the AR(2) model

$$y_t = \mu + a\, y_{t-1} + b\, y_{t-2} + u_t \,,$$

where the error term is white noise with variance σ_u^2. Assume the model is stationary.

a) Find (derive) the mean of y.
b) Find the first order autocovariance as a function of the variance. (It takes more work to find the variance, so you are not asked to do that.)

2. (20%) Consider the model

$$y = \mu + u \,,$$

where u is $\mathrm{N}(0, \sigma^2)$.

a) Write down the likelihood function for a sample of N observations. Assume the observations are independent.
b) Find the gradient vector and the outer product of gradients. (You only need to do this for one observation. I am asking for $G_i G_i'$ in the notation of the ML-note.)
c) Find the Hessian (the matrix of second derivatives). (Again, you only need to find this

for one observation.)

d) Show that the expectation of the outer product of gradients is equal to minus the expected value of the Hessian. (If you prefer, you can give a general proof, as in class, instead.)

3. (20%)

a) Explain what is meant by a censored regression model. You can use a figure here.

b) Write down the likelihood function for the censored regression model.

c) Assume you are estimating the model

$$Y_i = aX_i + u_i \ ,$$

by OLS. Here $a = 1$ is a scalar and we assume for simplicity that there is no intercept. In the true underlying model (not censured or truncated) the error term has mean 0.

Further assume that you only have 3 observations: $X' = 2, 4, 8$. Assume that the data are censured and values of Y larger than 6 is set to 6. Also assume that we know that the distribution of the innovation term u_i is such that it takes only the values -3 and 3 (each with probability 0.5).

d) Find the expected value of the OLS estimator of a.

4) (20%)

a) Write down the latent-variable model used to derive the Probit model and derive expression for the probability of each outcome.

b) Write down the log-likelihood function for the *Logit* model for a sample of N observations.

5. (30%) On the next pages, I have reproduced two pieces of Matlab code that you used for a homework.

a) (5%) Explain what Code I does (what model is being estimated?).

b) (10%) Fill in the missing line in Code I. (It does not have to be perfect Matlab code.)

c) (5%) Explain what Code II does (what model is being estimated?).

d) (10%) Explain what happens in Code II in the lines marked “ WHAT HAPPENS IN THIS LINE AND THE NEXT?” .

Code I

```
function [L] = logl_xx(b)
% The following is the loglikelihood function for an xx model.

global x z N

b0 = b(1);
b1 = b(2);
XB = b0*ones(size(x,1),1) + b1*x ;

L=0 ;
for i = 1:N
  if z(i) == 1
    L = L + log(normcdf(XB(i)));
  else
    L =FILL IN HERE;
  end
end
L = -L;

end
```

Code II

```
function L = logl_xxx( b )

% The following constructs the loglikelihood function for an xxx.

global x T

omega = zeros(T,T);                                                            % Placeholde
b0 = b(1);                                                                     % Mean.
b1 = b(2);                                                                     % xxx coeffi
s  = b(3);                                                                     % Standard d

omega = omega + eye(T).*(s^2).*(1+b1^2);

omega(2,1) = (s^2)*b1;                                                         omega(T-1,T)

for i = 2:T-1
    omega(i-1,i) = (s^2)*b1;         % WHAT HAPPENS IN THIS LINE AND THE NEXT?
    omega(i+1,i) = omega(i-1,i);
end

L = - 0.5*log(abs(det(omega)))- 0.5*(x-b0)'*inv(omega)*(x-b0);                 % Loglikelih
L = L - 0.5*T*log(2*pi);
L = -L;                                                                        % Negative o

end
```