Midterm Exam 1—Monday, September 27th, 2021. Five questions. Each sub-question in the following carries equal weight except if otherwise noted.

1. (15%) Assume you have estimated two parameters $\hat{\beta}_1$ and $\hat{\beta}_2$ and that the inverse of the variance-covariance matrix for these two parameters is

$$\Sigma^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} .$$

1) Write down the Wald test for the hypothesis $\{\beta_1, \beta_2\} = \{1, 2\}$. (This should be a scalar expression.)

2) Write down the Wald test for the hypothesis $\{\beta_1^2 - \beta_2, \beta_2\} = \{1, 2\}$. (This should be a scalar expression.)

2. (30%) Consider the CDF of exponential distribution, which is commonly used for duration modeling:
$$F(t) = 1 - \exp(-t/\theta) .$$
(For the exponential distribution, $EX = 1/\theta$ and $EX^2 = 1/\theta^2$.)

1) Find the density, survival function, and hazard function.

2) Find the asymptotic information "matrix" analytically by taking second-order derivatives and explain how you can estimate the variance of your estimate using this. (You only need to state the variance as a function of what you did, not derive the formula.)

3) Find the asymptotic information "matrix" analytically by means of first-order derivatives.

3. (15%) Assume you are estimating the model

$$Y_i = \beta X_i + u_i ,$$

by OLS. (Here β is a scalar and we assume for simplicity that there is no intercept and, in the true underlying model (not censured or truncated), the error term has mean 0.)

Assume that the model is truncated for values of $Y_i \leq K$. Derive the unbiased estimator of β. (If you cannot do it, you will get partial points for explaining the issues, maybe using a figure.)

4. (30%) Fill in the missing Matlab code (two places), using the symbols from the program (you can just write that line on your answer sheet).

```matlab
function [ L ] = logl_MA( b0 )
% Loglikelihood for MA(1) and MA(2).

global x T MA

omega = zeros(T,T);                                             % Placeholder fo
b1 = b0(1);                                                     % Mean.
s  = b0(2);                                                     % Standard devia
b2 = b0(3);                                                     % MA coefficient
mu = ones(T,1)*b1;                                              % Mean Vector.

    if MA == 1                                                  % For MA(1) pro

        omega = omega + eye(T).*(s^2).*(1+b2^2);                % Fill in the va

        omega(2,1) = FILL IN HERE;                                  % Fill in th
        omega(T-1,T) = omega(2,1);

        for i = 2:T-1
            omega(i-1,i) = (s^2)*b2;
            omega(i+1,i) = omega(i-1,i);
        end

    elseif MA == 2 ...skipped here                              %

L = -0.5*T*log(2*pi) - FILL IN HERE;

L = -L;                                                         % Negative of l

end
```

5. (10%) Fill in the missing Matlab code, using the symbols from the program (you can just write that line on your answer sheet).

```matlab
function L = logl_xxx(b)

% This is the loglikelihood function for xxx.

global x z

b0 = b(1);
b1 = b(2);

L = z.*log(1./(1+exp(-b0-x*b1))) + (1-z).*(SOMETHING MISSING);
L = -sum(L);

end
```