

**Final Exam—December 3, 2021**

Each sub-question in the following carries equal weight.

1. (15%) a) What does the Matlab code below estimate? Write down the model so it is clear what the parameters  $b_0, b_1, b_2$  capture.

**Final Exam Code 1.**

```
function L = logl_XX( b )

% The following constructs the loglikelihood function for xxxx

global x T

omega = zeros(T,T);
b0 = b(1);
b1 = b(2);
b2 = b(3);
omega = omega + eye(T).*(b2^2).*(1+b1^2);

omega(2,1) = (b2^2)*b1;
omega(T-1,T) = omega(2,1);

for i = 2:T-1
    omega(i-1,i) = (b2^2)*b1;
    omega(i+1,i) = omega(i-1,i);
end
```

% Placeholder  
% Mean.  
% xxx coef

```

L = - 0.5*log(abs(det(omega)))- 0.5*(x-b0)'*inv(omega)*(x-b0);
L = L - 0.5*T*log(2*pi);
L = -L;

end

```

% Loglikel

% Negative

2. (15%) a) What does the Matlab function "funct" below do? Explain what is the output and what is  $K1$  and what is  $\omega$ .

### Final Exam Code 2.

```

function W = funct(b,mat)

global lag n T

f = orth(b);      % f is a vector of orthogonality conditions

K1 =round(1.1447*(a1*(T-lag-1))^(1/3))

omega = f'*f;

for i = 1:K1
    omegai = f(i+1:T-lag-1,:)'*f(1:T-lag-1-i,:);
    omega = omega + (1-(i/K1))*(omegai+omegai');
end

omega = omega./(T-lag-1);

end

end

```

3. (20%) a) What does the Matlab code below estimate?
- b) How would you test  $a_1 - b_1 = 0$ ? Suggest at least one method, but you need to be explicit. (For example, if you want to involve a variance, you have to explain in words how you would estimate it.)

### Final Exam Code 3.

```
function [L] = logl_ss(b)

global x w y z N

b0 = b(1);
b1 = b(2);
sigmau = b(3);
g0 = b(4);
a1=b(5)

XB = b0*ones(size(x,1),1) + b1*x ;
WG = g0*ones(size(w,1),1) + a1*x+w ;

L=0 ;

for i = 1:N
    if z(i) == 1
        L = L + log((1/sigmau)*normpdf((y(i) - XB(i))/sigmau))...
            + log(normcdf(WG(i)));
    elseif z(i) == 0
        L = L + log(normcdf(-WG(i)));
    end
end

L = -L;
end
```

**4. (24%)** a) Assume you have estimated 3 parameters, with estimates  $\hat{\beta}_1$ ,  $\hat{\beta}_2$ , and  $\hat{\beta}_3$ , and assume that you know for sure the estimates are normally distributed and the variance-covariance matrix is

$$\Sigma = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{pmatrix} .$$

- a) Write down a (Wald) test statistic for  $\beta_2 + \beta_3 = 2$ . (I want you to write this as a scalar.)
- b) What is the distribution of the test statistic you write down?
- c) Write down a test statistic for  $(\beta_2, \beta_3) = (2, 1)$ . (I want you to write this as a scalar also.)
- d) What is the distribution of the test statistic you write down?
- e) Write down a test statistic for  $\beta_2^2 - \beta_3 = 2$ . (I want you to write this as a scalar also, the term should include one or more of the  $\hat{\beta}_i$ .)
- f) Assuming the coefficient are consistent, what is the asymptotic distribution of the test statistic you write down?

**5. (16%)** a) Explain what is meant by a truncated linear model.

b) Show in a figure why the truncated linear model is biased. Explain the direction of bias for your setup.

c) Derive the unbiased maximum likelihood estimator for the truncated model.

**6. (10%)** Write down a linear model for which an econometrician would want to use clustered standard errors. Make an explicit example and explain what the robust clustered standard error estimator does. For this short question, you do not necessarily have to use mathematical formulas.