ECONOMETRICS II, Fall 2022
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Final Exam. December 2. 7 questions.

1. (14%) a) Consider the AR(2) model

$$y_t = 2 + .1y_{t-1} + .9y_{t-2} + u_t \,,$$

where the error term is white noise. Is the lag-polynomium consistent with y_t being stationary?
b) Consider the ARMA(1,1) model

$$y_t = 2 + y_{t-1} + u_t - u_{t-1} \,.$$

Could y_t be stationary in this model?

2. (14%) a) Give an example of a linear model with truncated data (the dependent variable truncated) and explain the direction of bias.
b) Write down the log-likelihood function that provides a consistent estimator for the linear model with truncated data.

3. (25%) a) Assume you have estimated 3 parameters, with estimates $\hat{\beta}_1 = 7$, $\hat{\beta}_2 = 9$, and $\hat{\beta}_3 = 11$, and assume that you know for sure the estimates are normally distributed and the variance-covariance matrix is

$$\Sigma = \begin{pmatrix} 5 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{pmatrix} .$$

a) Write down a (Wald) test statistic for $\beta_2 + \beta_3 = 2$. (I want you to write this as a scalar, a real number, although you do not have to solve for ratios or invert matrices in the questions here.)
b) What is the distribution of the test statistic you wrote down?
c) Write down a test statistic for (β_2, β_3)=(2,1). (I want you to write this as a scalar also.)
d) What is the distribution of the test statistic you wrote down?
e) Write down a test statistic for the hypothesis $\log(\beta_1) - \beta_3^2 = 2$.
f) Assuming the coefficients are consistent, what is the asymptotic distribution of the test statistic you wrote down?

4. (14%) a) What does the Matlab code below do. Please write down the equations being estimated and explain the logic.

b) What is the coefficient ρ capturing?

c) Explain how you could use simple modifications of this program to test the hypothesis $b1 = g1$.

Code

```
function [L] = logl_ss(b)

global x w y z N

b0 = b(1);
b1 = b(2);
sigmau = b(3);
g0 = b(4);
g1 = b(5);
rho = b(6);

XB = b0*ones(size(x,1),1) + b1*x ;
WG = g0*ones(size(w,1),1) + g1*w ;

L=0 ;

for i = 1:N
  if z(i) == 1
    L = L + log((1/sigmau)*normpdf((y(i) - XB(i))/sigmau))...
   +log(  normcdf( (1/sqrt(abs(1-rho^2)))*(WG(i) + (rho/sigmau)*(y(i) - XB(i))) )  );
  elseif z(i) == 0
    L = L + log(normcdf(-WG(i)));
  end
end

L = -L;

end
```

5. (12%) What is the Augmented Dickey-Fuller test for testing a unit root with drift against a stationary model with trend? What regression would you run, what is the null hypothesis? Explain why the regression null is a test of unit root. I am not asking for derivations.

6. (9%) Consider a panel data model with fixed effects.
Show, using Frisch-Waugh, how you can remove the fixed effect and then estimate a "corrected data" model without fixed effects (dummies) by OLS.

7. (12%) In the code below $xhet(i + ((t-1) * N), :)$ is the regressors for a subgroup of the data and resid are the residuals from an initial regression.
a) Write down in regular matematical symbols what the code does.
b) Why are we doing this? Explain what the purpose is.

```
W = zeros(numx,numx);                                                    %

for i = 1:N

    uX = zeros(1,numx);

    for t = 1:T

        uX = uX + resid_het(i+((t-1)*N))*xhet(i+((t-1)*N),:);        % .

    end

    W = W + uX'*uX;

    clear xxxx

end

W = W*N*(nobs-1)/((nobs-numx)*(N-1));
```