ECONOMETRICS 1, Spring, 2016
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Midterm Exam - April 8, 2016

Each sub-question in the following carries equal weight.

1. (20%) Assume that you are interested in estimating the model

$$Y_i \ = \ \beta_0 \ + \ \beta_1 X_{1i} \ + \ \beta_2 X_{2i} \ + \ \epsilon_i$$

by OLS. Assume that you have 100 observations and that you know that

$$var(X_1) = 3, \ \ var(X_2) = 1, \ \ cov(X_1, X_2) = 0, \ \ cov(X_1, Y) = 5, \ \ cov(X_2, Y) = 4, \ \ var(Y) = 30 \ .$$

Here, $var(X_1)$ is short-hand for $\frac{1}{N}\Sigma_{i=1}^{N}(X_i - \bar{X}_i)^2$ and $cov(X_2, Y)$ is short-hand for $\frac{1}{N}\Sigma_{i=1}^{N}(X_i - \bar{X}_i)(Y_i - \bar{Y}_i)$ for any X and Y.

a) Find the estimated coefficients b_1 and b_2.
b) Find R^2 and $\hat{\sigma}^2$.
c) Perform a 5% one-sided t-test for the hypothesis $\beta_1 > 1$. (Use the critical value in the attached table.) (If you could not find $\hat{\sigma}^2$ in b) use a value of 2.0).
d) Construct a 95% confidence interval for β_2.

2. (20%) Assume that you are looking at a standard linear model with 5 regressors (including the constant) and that you estimate the model over a period of length 100 and that you find the sum of squared residuals to be equal to 70.
a) Assume that you suspect the last 10 periods are different and you therefore estimate the model using the first 90 observations where you obtain a sum of squared residuals equal to 40 and you estimate the model using the last 10 observations and obtain a sum of squared residuals equal to 5. Test whether the parameters of the last 10 periods are equal to the parameters of the first 90 periods. (State the assumptions under which the test is valid).
b) Now assume that you only suspect the last 2 periods to be different and that you estimate the model for the first 98 periods and obtain a sum of squared residuals equal to 55. Now (under the assumptions that you used in part a)) test whether the model were unchanged for the last two periods.

3. (20%) Prove that the t-value is t-distributed under the standard assumptions, including normality, of the linear regression model.

4. (10%) Assume that you have estimated (from a sample of individuals) the wage equation

$$W_i \;=\; 10 \;+\; 0.2EXP_i \;-\; .001EXP2_i \;+\; .05D_i \;+\; .002DEXP_i$$

by OLS, where W_i is earnings, EXP_i is labor market experience, D_i is a dummy that is one if individual i is a male and 0 otherwise and $DEXP_i$ is the product of D_i and EXP_i.
What is the predicted earnings for a male with 10 years of experience?

5. (15%) Assume that you are interested in estimating the model

$$Y_i \;=\; \beta_0 \;+\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \beta_3 X_{3i} \;+\; \epsilon_i$$

by OLS using 100 observations. Assume that you estimate the model in the form

$$(*) \qquad Y_i \;=\; \beta_0 \;+\; \beta_1 X_{1i} \;+\; \beta_2 \tilde{X}_{2i} \;+\; \beta_3 \tilde{X}_{3i} \;+\; \epsilon_i$$

where (using partitioned matrix notation)

$$[\tilde{X}_2 \tilde{X}_3] \;=\; (I - P_{X_1})[X_2 X_3] \; .$$

Assume that you have estimated (*) and obtained some estimated b_1, b_2, b_3.
a) Assume you regress Y on X_1, $\tilde{X}_2$, and $\tilde{X}_3$. Call the estimated coefficients γ_1, γ_2 and γ_3. State, for $i = 1, 2$, and 3, the conditions (if any) under which $\gamma_i = b_i$.
b) Assume that you now regress Y on X_1 and $\tilde{X}_2$. Will you (in general) get the same coefficient (b_1) to X_1? And would you obtain the same estimated coefficient (b_2) to X_2?

6. (15%) Assume that you again are interested in estimating the model

$$Y_i \;=\; \beta_0 \;+\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \epsilon_i$$

by OLS. Assume that the standard assumptions for the linear model holds except that $E[\epsilon_i] = 3$. Find the expected value of the OLS estimators b_1, b_2 and b_3. And the expected value of $\hat{\sigma}^2$.