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Dynamic Macroeconomics. Final December 2, 2011.
(4 questions.)

1. (30%) Consider an economy with $2N$ households of two different types called "Odd" and "Even." There are N of each type. The income process for Even is $Y_t^o = 1$, if t is even—i.e., (0,2,4,...) and 0 otherwise—and the income for Odd is $Y_t^o = 1$, if t is odd—i.e., (1,3,5,...) and 0 otherwise. All consumers maximize $U = \Sigma_{t=0}^{\infty} \beta^t u(c_t)$ where $u()$ is increasing, concave, and satisfies an Inada condition at 0.
A household takes the price sequence q_t^0 as given and chooses a consumption sequence to maximize $\Sigma_{t=0}^{\infty} u(c_t)$ subject to the budget constraint

$$\Sigma_{t=0}^{\infty} q_t^0 c_t = \Sigma_{t=0}^{\infty} q_t^0 y_t \ .$$

Question a. Find the first-order conditions for the household's problem.

Definition 1: A competitive equilibrium is a price sequence $\{q_t^0\}_{t=0}^{\infty}$ and an allocation $\{c_t^o, c_t^e\}_{t=0}^{\infty}$ that have the property that (a) given the price sequence, the allocation solves the optimum problem of households of both types; and (b) $c_t^o + c_t^e = 1$ for all $t \geq 0$.

To find an equilibrium, we have to produce an allocation and a price system for which we can verify that the first-order conditions of both households are satisfied. We start with a guess inspired by the constant-consumption property of the Pareto optimal allocation. We guess that $c_t^o = c^o, c_t^e = c^e, \forall t$, where $c^e + c^o = 1$.

Question b. Show that this guess and the first-order condition for the odd agents imply

$$q_t^0 = q_0^0 \beta^t \ , \quad (*)$$

where we are free to normalize by setting $q_0^0 = 1$, and find the consumption of each of the two types of agents?

2. (40%) Assume f_t is a vector of random variables which have expectation 0.

Question a. Outline how the variance matrix

$$\Omega = \lim_{J\to\infty} \sum_{j=-J}^{J} E[f_t f'_{t-j}]$$

can be estimated non-parametrically.

Question b. What are the weights (you have to make it clear what is being weighted) for the Bartlett/Newey-West/"tent" estimator of the optimal weighting matrix?

Question c. What are the rates at which the "bandwidth" goes to infinity with T (the number of observations) for the Bartlett kernel and the Quadratic Spectral kernel, respectively?

Question d. Outline the idea of "pre-whitening."

3. (20%) An unemployed worker samples wage offers on the following terms: Each period, with probability $\phi, 1 > \phi > 0$, he or she receives no offer (we may regard this as a wage offer of zero forever). With probability $(1-\phi)$ he or she receives an offer to work for w forever, where w is drawn from a cumulative distribution function $F(w)$. Successive draws across periods are independently and identically distributed. The worker chooses a strategy to maximize

$$E\Sigma_{t=0}^{\infty}\beta^t y_t \ ,$$

where $0 < \beta < 1$, $y_t = w$ if the worker is employed, and $y_t = c$ if the worker is unemployed. Here c is unemployment compensation, and w is the wage at which the worker is employed. Assume that, having once accepted a job offer at wage w, the worker stays in the job forever. Let $v(w)$ be the expected value of $\Sigma_{t=0}^{\infty}\beta^t y_t$ for an unemployed worker who has offer w in hand and who behaves optimally.

Question a. Write the Bellman equation for the worker's problem.

4. (10%) **Question a.** Sketch the model of the shopping time monetary economy. You don't need to derive first order conditions but give the intuition of the model with a focus on the shopping (transaction) technology. What do we need to assume about this technology (like signs of the derivatives with respect the arguments)?