Asset Pricing

Chapter 13: Ljungqvist and Sargent

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Introduction

First, we begin with an approach that -

- Uses only the Euler equations for a maximizing consumer
- Does not specify a complete general equilibrium model

Later, use complete market approach also

Introduction

Overview

- Asset Euler Equations
- Martingale Theories of Consumption and Stock Prices
- Equilibrium Asset Pricing
- Stock Prices without Bubbles

Asset Euler Equations

The optimization problem of a single agent and trade in two assets.

The agent with wealth $A_t > 0$ maximize expected lifetime utility,

$$E_t \sum_{j=0}^{\infty} \beta^j u\left(c_{t+j}\right), \qquad 0 < \beta < 1, \qquad (13.2.1)$$

- E_t is expectation conditional on information known at time t.
- U(.) is concave, strictly increasing, and twice continuously differentiable.

Agent transfers wealth over time through bond and equity holdings to finance future consumption.

Asset Euler Equations

Define:

R_t : risk-free real gross interest rate in one-period bonds

L_t : gross payout on bond holdings between periods *t* and *t+1*

s_t : holding of equity shares between periods *t* and *t+1*

p_t : share price in period *t* net of that period's dividend

y_t : stochastic dividend stream from equity at *t*

The budget constraint is then,

$$c_t + R_t^{-1} L_t + p_t s_t \leq A_t, \qquad (13.2.2)$$

and next period's wealth is,

$$A_{t+1} = L_t + (p_{t+1} + y_{t+1}) \, s_t. \qquad (13.2.3)$$

Asset Euler Equations

A dynamic programming problem

state variables : A_t and current and past *y*

controls : L_t and s_t

At interior solutions, the Euler equations are

$$u'(c_t) R_t^{-1} = E_t \beta u'(c_{t+1}), \tag{13.2.4}$$

$$u'(c_t) p_t = E_t \beta (y_{t+1} + p_{t+1}) u'(c_{t+1}). \tag{13.2.5}$$

Optimal solution must also satisfy the transversality conditions

$$\lim_{k \to \infty} E_t \beta^k u'(c_{t+k}) R_{t+k}^{-1} L_{t+k} = 0, \tag{13.2.6}$$

$$\lim_{k \to \infty} E_t \beta^k u'(c_{t+k}) p_{t+k} s_{t+k} = 0. \tag{13.2.7}$$

That is, agent neither dies with positive asset holding nor can die accumulating debts.

Martingale Theories of Consumption and Stock Prices

By making special assumptions about either R_t or *u'(c)* in Euler equations.

First, assume that risk-free interest rate is constant over time, $R_t = R > 1$

(13.2.4) ⇒ $$E_t u'(c_{t+1}) = (\beta R)^{-1} u'(c_t), \qquad (13.3.1)$$

Robert Hall's (1978) result: MU of consumption follows univariate linear 1st order Markov process.

⇓

No other variables in the information set help to predict (to Granger cause) $u'(c_{t+1})$, once lagged $u'(c_t)$ has been included.

Hall tested for the absence of Granger causality from other variables to c_t for the special case of quadratic utility.

Martingale Theories of Consumption and Stock Prices

- Example

  With the CRRA utility function $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$, equation (13.3.1) becomes

$$(\beta R)^{-1} = E_t \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}$$

Martingale Theories of Consumption and Stock Prices

Efficient Stock Markets : Price of a stock follows a martingale process.

(13.2.5) ➡ $$E_t \beta \left(y_{t+1} + p_{t+1}\right) \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} = p_t$$

Using covariance formula,

$$\beta E_t \left(y_{t+1} + p_{t+1}\right) E_t \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} + \beta \operatorname{cov}_t \left[\left(y_{t+1} + p_{t+1}\right), \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}\right] = p_t. \quad (13.3.2)$$

To obtain a martingale theory of stock prices, need to assume

i) $E_t u'(c_{t+1})/u'(c_t)$ is a constant.

ii) $\operatorname{cov}_t \left[\left(y_{t+1} + p_{t+1}\right), \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}\right] = 0$

Martingale Theories of Consumption and Stock Prices

- very restrictive conditions and hold under very special circumstances, e.g. **risk neutral agent** i.e. $u(c_t)$ is linear in c_t so that $u'(c_t)$ is independent of c_t

(13.2.5) → $$E_t\beta\left(y_{t+1}+p_{t+1}\right)=p_t. \qquad (13.3.3)$$

i.e. adjusted for dividends and discounting, the share price follows a 1st order univariate Markov process and no other variables Granger causes the share price.

The stochastic difference equation (13.3.3) has the class of solutions

$$p_t=E_t\sum_{j=1}^{\infty}\beta^j y_{t+j}+\xi_t\left(\frac{1}{\beta}\right)^t, \qquad (13.3.4)$$

Share price p_t is the sum of discounted expected future dividends and a bubble term unrelated to any fundamentals.

Equilibrium Asset Pricing

Simple representative agent endowment economy: **Lucas Asset Pricing Model**

- a large number of identical agents
- preferences : (13.2.1)
- only durable good: a set of identical "trees" one for each person
- dividends (fruits) at the beginning of *t*: y_t
- fruit is not storable; tree is perfectly durable
- each agent starts life at time zero with one tree

All agents maximize **13.2.1** subject to budget constraints **13.2.2** and **13.2.3** and transversality conditions.

Equilibrium Asset Pricing

In equilibrium, asset prices clear the markets i.e.,

- $\sum$ bond holdings of all agents = 0,
- total stock = aggregate number of shares

Identical agents in terms of preferences and endowments: a representative agent model

<u>Steps:</u>

i. Given preferences, technology and endowments, solve for equilibrium intertemporal consumption allocation

Planner problem:

$$\text{maximize } E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \text{ subject to } c_t \leq y_t$$

Solution: $c_t = y_t$

Equilibrium Asset Pricing

ii. Set up a competitive market for assets, permit agents to buy and sell at equilibrium asset prices subject to constraints, and find an agent's Euler equations

*Euler equations: **13.2.4** and **13.2.5***

iii. Equate consumption in Euler equation to equilibrium consumption in planner problem, the risk-free interest rate and the share price are given by

$$u'(y_t) R_t^{-1} = E_t \beta u'(y_{t+1}), \tag{13.5.1}$$

$$u'(y_t) p_t = E_t \beta (y_{t+1} + p_{t+1}) u'(y_{t+1}). \tag{13.5.2}$$

Stock Prices without Bubbles

Using recursions and LIE, $E_t E_{t+1}(\cdot) = E_t(\cdot)$ on equation (13.5.2), the expression for the equilibrium share price is

$$u'(y_t)\,p_t = E_t \sum_{j=1}^{\infty} \beta^j u'(y_{t+j})\,y_{t+j} + E_t \lim_{k\to\infty} \beta^k u'(y_{t+k})\,p_{t+k}. \qquad (13.6.1)$$

Market clearing condition:

- agents must be willing to hold their endowments of trees forever

⇒ $E_t \lim_{k\to\infty} \beta^k u'(y_{t+k})\,p_{t+k} = 0$

so that, $u'(y_t)\,p_t = E_t \sum_{j=1}^{\infty} \beta^j u'(y_{t+j})\,y_{t+j}$

[MU gain of selling shares = MU loss of holding the asset forever and consuming the future stream of dividends]

Stock Prices without Bubbles

i. $u'(y_t)p_t > E_t \sum_{j=1}^{\infty} \beta^j u'(y_{t+j}) y_{t+j}$, agents would like to sell some of their shares → p_t falls

ii. $u'(y_t)p_t < E_t \sum_{j=1}^{\infty} \beta^j u'(y_{t+j}) y_{t+j}$, agents would like to purchase more share → p_t rises

Thus, the equilibrium price must satisfy

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{u'(y_{t+j})}{u'(y_t)} y_{t+j}, \qquad (13.6.2)$$

i.e., the share price is the sum of expected discounted stream of dividends but with time-varying and stochastic discount rates.

Computing Asset Pricing

<u>Example 1: Logarithmic preference</u>

$$u(c_t) = \ln c_t$$

Then equation (13.6.2) becomes

$$p_t = \frac{\beta}{1-\beta} y_t. \tag{13.7.1}$$

- Equation (13.7.1) is asset-pricing function, which maps the state of the economy at t, y_t, into the price of a Lucas tree at t.

<u>Example 2: Asset Pricing with growth</u>

Consider a Lucas tree in a pure endowment economy with $c_t = d_t$ and $d_{t+1} = \lambda_{t+1} d_t$, where λ_t is Markov with transition matrix P.

Computing Asset Pricing

For the CRRA utility $u(c) = c^{1-\gamma}/(1-\gamma)$, the (ex-dividend) price of the Lucas tree, p_t, satisfies,

$$p_t = E_t \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} (p_{t+1} + d_{t+1}) \right]$$

Dividing by d_t and rearranging,

$$\frac{p_t}{d_t} = E_t \left[\beta \left(\lambda_{t+1} \right)^{1-\gamma} \left(\frac{p_{t+1}}{d_{t+1}} + 1 \right) \right]$$

Questions?

Comments