Math Review: Calculating with Units

Most of the quantities we calculate in science have units of measurement that go with them. The formulas we use to do these calculations come in two flavors, those where we must input numbers into the formulas in specific units, and those where we can be a bit more flexible in the units we use for our inputs. To see the difference between the two types of formulas, lets take a look at an example of a formula of each type. First, lets look at Wien's law, a formula that relates the peak wavelength of a glowing object to its temperature:

$$\lambda_{max} T = 3 \times 10^6 \, nm \cdot K .$$

The second formula allows us to calculate the angular resolution of a telescope, that is, what is the smallest angle two stars could be separated by and have us still discern two stars and not a single merged image:

$$\theta = 3{,}000{,}000 \left(\frac{\lambda}{D} \right) Arc\,seconds$$

In this formula, λ is the wavelength of the light being observed, and D is the diameter of the telescope's front lens.

Notice in the first formula, the number on the right side has the specific units of nm time K. This means that if we knew the wavelength where the maximum amount of light emerged and wanted to calculate the temperature of the body, we *must use a wavelength in units of nm in our calculation,* this is the only way our answer for the temperature would come out in the standard units of K.

Now lets take a look at the second formula. This formula is designed to give us the angle in specific units (arcseconds in this case), and has inputs that have units. Notice in this case however, that both λ and D both are measured in length units, and that we divide one by another in the formula. This means that as long as the units of D and λ are the same, the units will cancel in the final calculations. That is, we could put in length units of nm, m, even furlongs if we wished, as long as we used the same units for both λ and D.

Test Yourself

A star that gives off its maximum amount of light at a wavelength of 1.79×10^{-7} feet is to be observed at that wavelength with a telescope that has a front lens with a diameter of 2 feet. If we wanted to calculate the temperature of the star in K would need to make unit conversions? What if we wanted to calculate the angular resolution of the telescope in arcseconds?

Answer: For the temperature we would need to convert feet to nanometers. For the angular resolution, both λ and D are in the same units (feet in this case), so no unit conversion is necessary.