Math Review: Taking Things to Powers

Sometimes the formulas we use in astronomy involve terms to different powers. For example, Kepler's third law for planets orbiting the Sun is

$$P^2 = a^3$$

where P is the time for the planet to orbit the Sun once in units of years, and a is the semimajor axis of the planet's orbit in astronomical units (AU). What if I told you that a planet had a semimajor axis of 2 AU, and asked you what the period of the orbit was? The formula you need to use involves P to the 2nd power, not P itself. This means that you somehow have to manipulate the formula in order to get a new formula that does involve P. The trick is to take the both sides of the formula to some new power (lets call it x) that will result in having just P, not P^2.

$$(P^2)^x = (a^3)^x$$

But what value of x is the right one? There two things you need to know in order to figure out what the correct value of x is. The first is that any quantity that we can write in a formula without being to any power can also be written as that thing to the 1st power, so we really just need a value of x that will give us P to the 1st power. The second is that if you have a quantity that you first raise to the x power, and then to the y power, then that is the same as the quantity being to the x times y power. As an example consider raising P to the power 2, followed by the power 3

$$(P^2)^3 = P^{2 \cdot 3} = P^6.$$

In the above example, what we want for x is then 2x=1, or x=1/2, so that the correct solution in this case is:

$$(P^2)^{1/2} = (a^3)^{1/2}$$

or

$$P = a^{3/2} = a^{1.5}$$

where the last form may be more convenient for punching into your calculator.

Your Turn

1. What if power would you need to take both sides of Kepler's law to if we wanted to solve for a, not P?
Answer: 1/3.

2. If you have the formula $L = M^{-2.5}$ and wanted to solve for M, what power would you need to take both sides of the formula to?
Answer: -0.4.