# Math Review: Comparison

Often in astronomy, we want to compare one value to another. For example, we might ask, "how would the luminosity of a star that was twice the temperature of the Sun, but only had a radius 1/3 the Sun, compare to that of the Sun?" To answer this question, we first need the formula that describes how the luminosity of a star depends on the radius and temperature of the star. When you study radiation and matter, you will find that this formula is:

$$L = 4\pi R^2 \sigma T^4$$

where L is the luminosity, R is the radius of the star, T is the surface temperature, and $\pi$ and $\sigma$ constants (that is, numbers that don't change).

For many students, the temptation is to look up all the relevant numbers, and punch them into their calculators for both the Sun and the other star. It turns out that this isn't really the best way to tackle the problem, for at least two reasons.

1.) If you use the calculator approach, you get two very large numbers; numbers that it is difficult to make sense of. For example, if I told you a certain star had a luminosity of $3.8 \times 10^{25} J/s$, you wouldn't have any sense in real world terms what that meant, but if instead I told you that the star had a luminosity of about 0.1 times that the Sun, you have some context that makes the number more meaningful for you.

2.) To use our calculator in this way, we need to know all kinds of numbers, numbers that we may or may not have handy.

Because of this, we normally tackle problems like this in a different way than simply grabbing our calculator. The first thing we do is go ahead and write down the formula we'd use for the Sun, substituting in variables for the radius and temperature of the Sun.

$$L_{Sun} = 4\pi R_{Sun}^{\;2} \sigma T_{Sun}^{\;4}$$

Now, you might think to yourself, "this hasn't really helped at all, we've just renamed L, R, and T!" This is true, but the real trick is now to do the same thing for the luminosity of the other star. That is, we'll substitute $R = 1/3R_{Sun}$, and $T = 2T_{Sun}$ in the formula for the other star's luminosity. If we do this, the formula for the luminosity of the star will look like:

$$L_{Star} = 4\pi (1/3R_{Sun})^2 \sigma (2T_{Sun})^4$$

The thing to notice is that this formula now has all the parts of the formula for the Sun's luminosity, namely 4, $\pi$, $\sigma$, $R_{Sun}$, and $T_{Sun}$. This means that if we just do a little algebra to get these parts by themselves, we can replace them with $L_{Sun}$. Let's see how this goes:

$$L_{Star} = 4\pi(1/3R_{Sun})^2\sigma(2T_{Sun})^4 = 4\pi(1/3)^2 R_{Sun}{}^2\sigma(2)^4 T_{Sun}{}^4 = (1/3)^2(2)^4\, 4\pi R_{Sun}{}^2\sigma T_{Sun}{}^4$$

Notice, the last bit is just $L_{Sun}$, so we can make that replacement!

$$L_{Star} = (1/3)^2(2)^4 L_{Sun}$$

What do we do about the numbers left in front of $L_{Sun}$? That combination of numbers is just the number of times brighter or dimmer that the star is compared to the Sun. We just need to work out the different powers and combine the numbers:

$$L_{Star} = (1/3)^2(2)^4 L_{Sun} = (1/9)(16)L_{Sun} = 16/9L_{Sun},$$

so the star will be 16/9, or roughly 2 times as bright as the Sun.

## Your Turn

How would a star that was twice the radius and 1/3 the temperature of the Sun compare to the Sun?

Answer: The star would only 4/81 times, or roughly 1/20th as bright as the Sun.

How much more kinetic energy does a car have when traveling at 70 miles/hour, compared to when it is traveling 30 miles/hour? The kinetic energy is given by $KE = \frac{1}{2}mv^2$, where m is the mass of the object, and v is its speed.

Answer: 49/9, or approximately 5.3 times as much kinetic energy.